TECHNISCHE UNIVERSITÄT DRESDEN

Computational Logic ∴ Group

FernUniversität in Hagen

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Semantic Characterizations of AGM Revision for Tarskian Logics

RuleML+RR, September 2022

Belief Revision

Belief revision: incorporating new information into agent's belief(s) consistently with minimal change.

$\mathcal{K} = \{p \land q\}$

$\mathcal{K}' = \{\neg q\}$

How to add $\mathcal{K}'$ to $\mathcal{K}$ so that the revision result ($\mathcal{K} \circ \mathcal{K}'$) is still consistent?

The symbol $\circ$ is a change operator.

AGM Postulates

Postulates by Alchourrón, Gärdenfors, and Makinson [AGM85] (generalized):

(G1) $\mathcal{K} \circ \Gamma \models \Gamma$.

(G2) If $\mathcal{K} \cup \Gamma$ is consistent, then $\mathcal{K} \circ \Gamma \equiv \mathcal{K} \cup \Gamma$.

(G3) If Γ is consistent then $\mathcal{K} \circ \Gamma$ is consistent.

(G4) If $\mathcal{K}_1 \equiv \mathcal{K}_2$ and $\Gamma_1 \equiv \Gamma_2$ then $\mathcal{K}_1 \circ \Gamma_1 \equiv \mathcal{K}_2 \circ \Gamma_2$.

(G5) $(\mathcal{K} \circ \Gamma_1) \cup \Gamma_2 \models \mathcal{K} \circ (\Gamma_1 \cup \Gamma_2)$.

(G6) If $(\mathcal{K} \circ \Gamma_1) \cup \Gamma_2$ is consistent, then $\mathcal{K} \circ (\Gamma_1 \cup \Gamma_2) \models (\mathcal{K} \circ \Gamma_1) \cup \Gamma_2$.

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Postulates $\Longleftarrow$ Operators $\Longrightarrow$ Construction

Motivation

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Our approach: Semantic characterization for (arbitrary) Tarskian logics, i.e. logics satisfying monotonicity:

$$\text{if } \mathcal{K}_1 \models \varphi \text{ and } \mathcal{K}_1 \subseteq \mathcal{K}_2, \text{ then } \mathcal{K}_2 \models \varphi,$$

e.g. propositional logic, first-order and second-order predicate logics, modal logics, and description logics.

Base Logic and Change Operator

Base Logic, Base Change Operator

A base logic is a quintuple $\mathbb{B} = (\mathcal{L}, \Omega, \models, \mathfrak{B}, \Cup)$, where

- $(\mathcal{L}, \Omega, \models)$ is a **Tarskian logic**,
- $\mathfrak{B} \subseteq \mathcal{P}(\mathcal{L})$ is a family of sets of sentences, called **bases**, and
- $\Cup : \mathfrak{B} \times \mathfrak{B} \to \mathfrak{B}$ satisfies $[\![\mathcal{B}_1 \Cup \mathcal{B}_2]\!] = [\![\mathcal{B}_1]\!] \cap [\![\mathcal{B}_2]\!]$ (**abstract union**).

A (multiple) base change operator for $\mathbb{B}$ is a function $\circ : \mathfrak{B} \times \mathfrak{B} \to \mathfrak{B}$.

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$\mathfrak{B}$ is a generalization for many bases settings:

- Arbitrary sets
- Finite sets
- Belief sets
- Single sentences

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- faithfulness-conditions for $\preceq_{(.)}$:
  - (F1) If $\mathcal{I}, \mathcal{I}' \models \mathcal{K}$, then $\mathcal{I} \prec_{\mathcal{K}} \mathcal{I}'$ does not hold.
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Representation Theorem [KM91]

In propositional logic with finite signature, a base change operator $\circ$ satisfies (G1)–(G6) if and only if $\circ$ is compatible with some **faithful preorder** assignment.

KM Theorem in $\mathbb{PL}_n$ - Example

- Let $\mathcal{K} = \{p \land q\}$ and $\mathcal{K}' = \{\neg q\}$. What is the result of $\mathcal{K} \circ \mathcal{K}'$?
- We have $\Omega = \{\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3, \mathcal{I}_4\}$, where

$\mathcal{I}_1$: pq

$\mathcal{I}_2$: $p\bar{q}$

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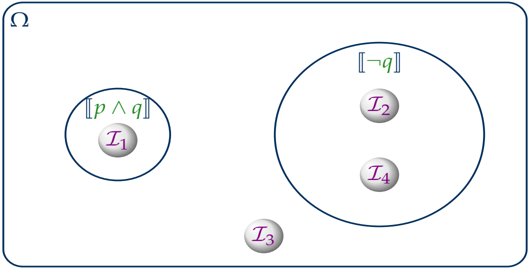

KM Theorem in $\mathbb{PL}_n$ - Example

- Let $\mathcal{K} = \{p \wedge q\}$ and $\mathcal{K}' = \{\neg q\}$. What is the result of $\mathcal{K} \circ \mathcal{K}'$?
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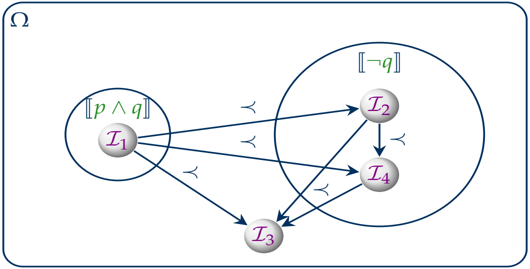

- Assume we have a faithful preorder assignment s.t. $\preceq_{p \wedge q}$: $\mathcal{I}_1 \prec \mathcal{I}_2 \prec \mathcal{I}_4 \prec \mathcal{I}_3$

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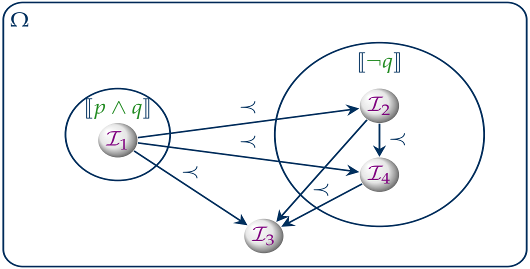


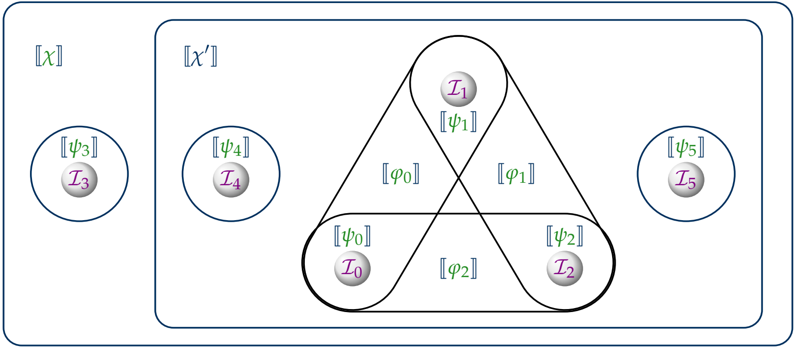
- Assume we have a faithful preorder assignment s.t. $\preceq_{p\land q}: \mathcal{I}_1 \prec \mathcal{I}_2 \prec \mathcal{I}_4 \prec \mathcal{I}_3$
- In this case, $min([\![\mathcal{K}']\!], \preceq_{p\land q}) = min([\![\neg q]\!], \preceq_{p\land q}) = \{\mathcal{I}_2\}$
- $[\![\mathcal{K} \circ \mathcal{K}']\!] = min([\![\mathcal{K}']\!], \preceq_{p\land q}) = \{\mathcal{I}_2\}$
- $\mathcal{K} \circ \mathcal{K}' = \{p \land \neg q\}$

KM representation theorem is a solid and inspiring semantic characterization of belief revision operator, yet requires further extension for logics beyond propositional logic.

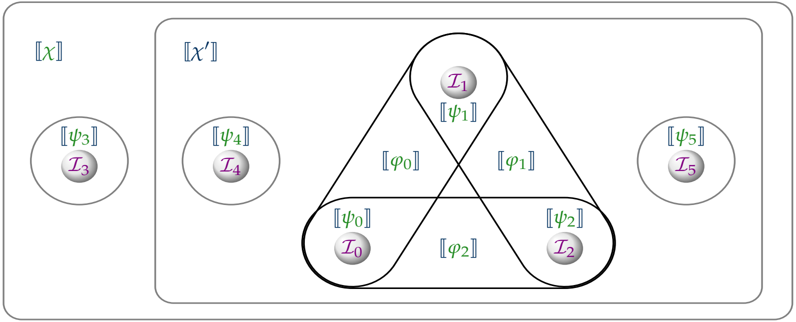
Tarskian Logic Example - $\mathbb{L}_{\text{Ex}}$

$[\![\psi_i]\!] = \{\mathcal{I}_i\}$ $\quad$ $[\![\chi]\!] = \{\mathcal{I}_0, \ldots, \mathcal{I}_5\}$ $\quad$ $[\![\varphi_1]\!] = \{\mathcal{I}_1, \mathcal{I}_2\}$

$[\![\chi']\!] = \{\mathcal{I}_1, \ldots, \mathcal{I}_5\}$ $\quad$ $[\![\varphi_0]\!] = \{\mathcal{I}_0, \mathcal{I}_1\}$ $\quad$ $[\![\varphi_2]\!] = \{\mathcal{I}_2, \mathcal{I}_0\}$



Example (cont.) - An AGM Revision Operator



Let $\circ_{\mathrm{Ex}}$ be the base change operator defined for $\mathcal{K}_{\mathrm{Ex}} = \{\psi_3\}$ such that (in particular):

- $\mathcal{K}_{\mathrm{Ex}} \circ_{\mathrm{Ex}} \{\chi\} = \{\psi_3, \chi\}$
- $\mathcal{K}_{\mathrm{Ex}} \circ_{\mathrm{Ex}} \{\chi'\} = \{\psi_4, \chi'\}$
- $\mathcal{K}_{\mathrm{Ex}} \circ_{\mathrm{Ex}} \{\varphi_0\} = \{\psi_0, \varphi_0\}$
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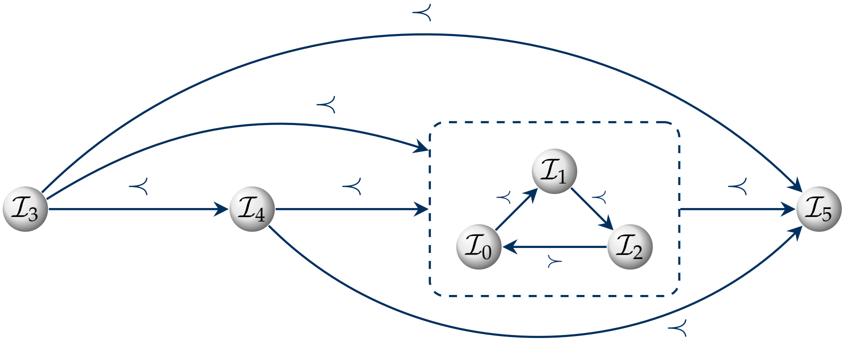
Running Example (cont.) - Problem in General Logic

- $\circ_{\mathrm{Ex}}$ satisfies postulates (G1)–(G6). Compatible relation:



Problem with Transitivity

- Transitivity and AGM Revision is incompatible in certain logics.
- Example: Horn logic requires cycles [DP15].
- Weaker property:

min-retractive

$\preceq$ is called min-retractive if:

$$\text{for every } \Gamma \in \mathfrak{B} \text{ and } \mathcal{I}', \mathcal{I} \in [\![\Gamma]\!] \text{ with } \mathcal{I}' \preceq \mathcal{I} \text{ and } \mathcal{I} \in \min([\![\Gamma]\!], \preceq)$$

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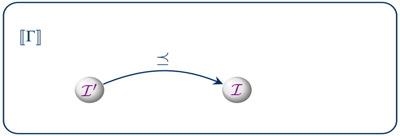

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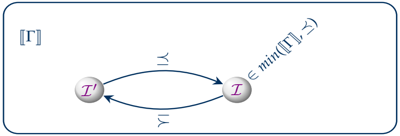

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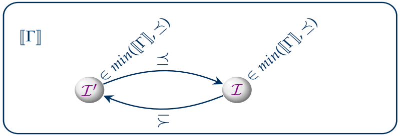

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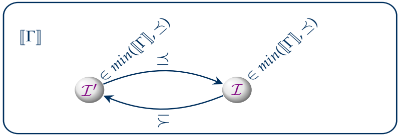

- Transitivity implies min-retractivity.

Problem with The Existence of Minimal Models

- Logics with infinitely many interpretations
- Example: first-order logic
- Minima are not guaranteed to exist [DPW18]

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$\preceq$ is called min-complete if:

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for every consistent $\Gamma \in \mathfrak{B}$ holds $\min(\llbracket\Gamma\rrbracket, \preceq) \neq \emptyset$.

- $\preceq$ is called min-friendly if it is min-retractive and min-complete.
- min-friendly assignment $\preceq_{(.)}$: every $\preceq_{\mathcal{K}}$ is min-friendly.

One-Way Representation Theorem for Tarskian Logics

Theorem

Let $\circ$ be a base change operator for some base logic $\mathbb{B}$. Then, $\circ$ satisfies (G1)–(G6) if and only if $\circ$ is compatible with some min-friendly faithful assignment.

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Is every min-friendly faithful assignment compatible with $\circ$?

Answer: No.

Two-Way Representation Theorem

Another problem: $\min(\llbracket\Gamma\rrbracket, \preceq)$ might not be the model set of any belief base [DPW18].

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Theorem

Let $\mathbb{B}$ be a base logic. Then the following hold:

- Every base change operator for $\mathbb{B}$ satisfying (G1)–(G6) is compatible with some min-expressible min-friendly faithful assignment.
- Every min-expressible min-friendly faithful assignment for $\mathbb{B}$ is compatible with some base change operator satisfying (G1)–(G6).

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Answer to the question: logics without critical loop. For instance: logics with disjunction.

Conclusion

- Semantically characterize AGM style belief base revision
  - KM-Style presentation
  - Applicable to arbitrary monotonic logics
- Critical loop identification in logics

Future work:

- Concrete realizations in popular KR formalisms: ontology languages
- Iterated belief revision [DP97]
- Relation with base postulates by Hansson [Han99]

Critical Loop

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In which **base logics**, every **AGM revision operator** is compatible with some faithful assignment that only yields **total preorders**?

Critical Loop

Let $\mathbb{B} = (\mathcal{L}, \Omega, \models, \mathfrak{B}, \Cup)$ be a base logic. Three or more bases $\Gamma_{0,1}, \Gamma_{1,2}, \ldots, \Gamma_{n,0} \in \mathfrak{B}$ are said to form a **critical loop of length** $(n+1)$ **for** $\mathbb{B}$ if there exists a base $\mathcal{K} \in \mathfrak{B}$ and consistent bases $\Gamma_0, \ldots, \Gamma_n \in \mathfrak{B}$ such that

(1) $[\![\mathcal{K} \Cup \Gamma_{i,i\oplus 1}]\!] = \emptyset$ for every $i \in \{0, \ldots, n\}$, where $\oplus$ is addition $\bmod (n+1)$,

(2) $[\![\Gamma_i]\!] \cup [\![\Gamma_{i\oplus 1}]\!] \subseteq [\![\Gamma_{i,i\oplus 1}]\!]$ and $[\![\Gamma_j \Cup \Gamma_i]\!] = \emptyset$ for each $i, j \in \{0, \ldots, n\}$ with $i \neq j$, and

(3) for each $\Gamma_\triangledown \in \mathfrak{B}$ that is consistent with at least three bases from $\Gamma_0, \ldots, \Gamma_n$, there exists some $\Gamma'_\triangledown \in \mathfrak{B}$ such that $[\![\Gamma'_\triangledown]\!] \neq \emptyset$ and $[\![\Gamma'_\triangledown]\!] \subseteq [\![\Gamma_\triangledown]\!] \setminus ([\![\Gamma_{0,1}]\!] \cup \ldots \cup [\![\Gamma_{n,0}]\!])$.

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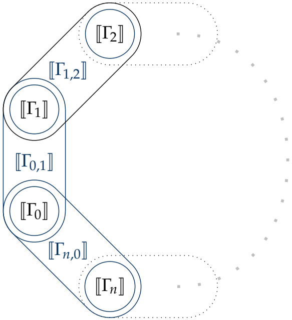

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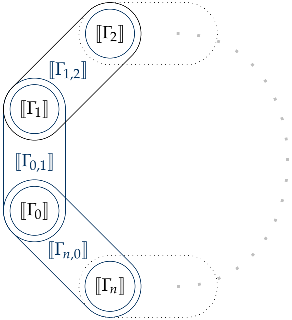
1. Allows construction of a circle
2. Does not touch the belief base $\mathcal{K}$



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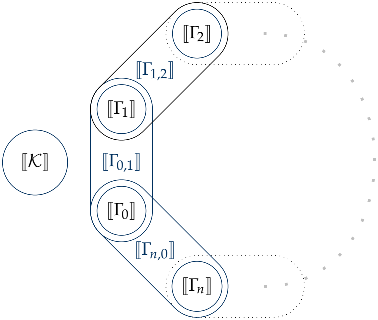

Critical Loops – In Pictures

Three Properties of a **critical loop** $\Gamma_{0,1}, \Gamma_{1,2}, \ldots, \Gamma_{n,0}$:

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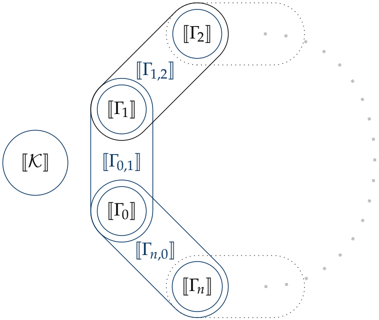

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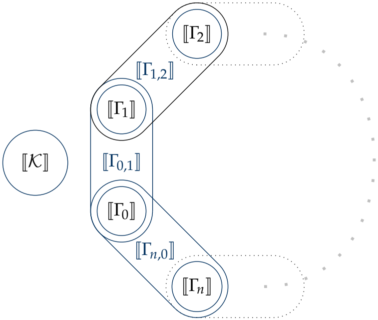

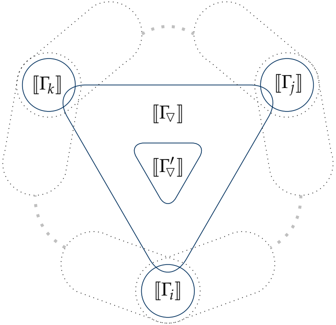

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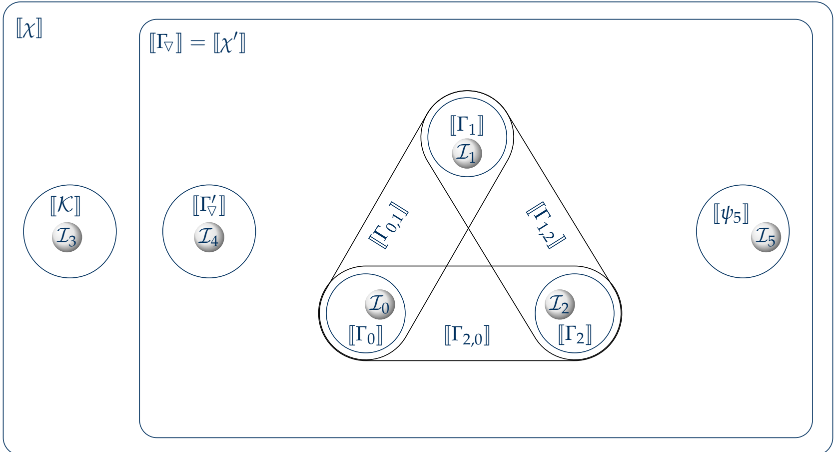

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