TECHNISCHE UNIVERSITÄT DRESDEN

RULE-BASED PARADIGMS IN KNOWLEDGE REPRESENTATION

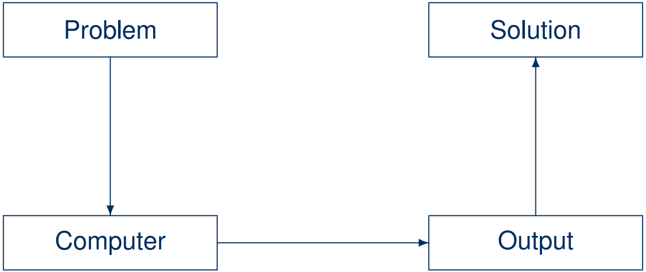
Seminar-Session 2: Declarative Problem Solving with Rules

Stefan Ellmauthaler

TU Dresden, 20th October 2021

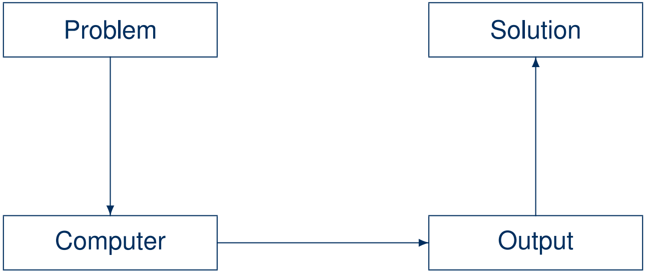
Outline

Computer Science



Computer Science

What is the problem? versus **What solves the problem?**



Traditional Programming

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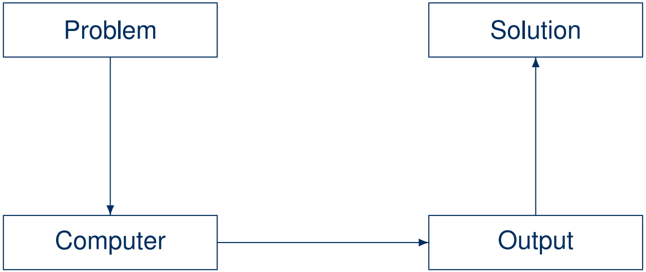

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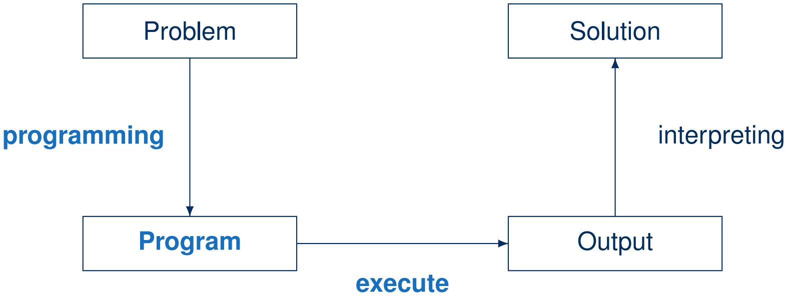

Declarative Problem Solving

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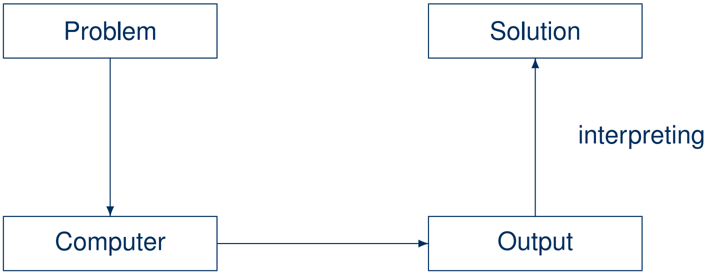

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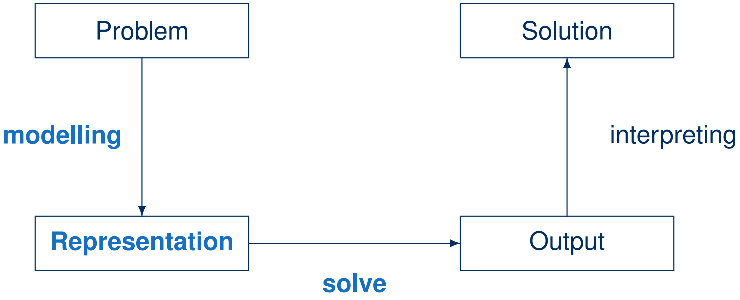

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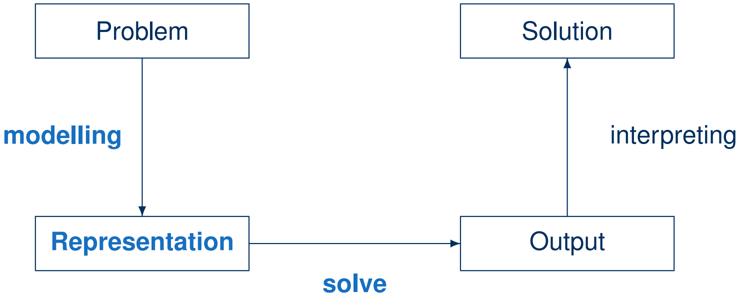

Why Declarative Rule-Based Problem Solving?

- Model problems . . .
  and use generalised and generic Algorithms to solve them
- Based on Logic and Computer Science
- Sub-field of Knowledge Representation
- Basic Idea (placatory):
  Human describes Problem, Computer solves Problem
- Rules are an intuitive way model problems

Definite Logic Programs: Syntax

Definition 2.1: Syntax of rules:

$$A \leftarrow B_1, \ldots, B_n$$

where A and the B_i are ground atoms.

- A is called head
- $B_1, \ldots, B_n$ is the body of the rule
- facts are rules where $n = 0$; "$\leftarrow$" may be omitted

Definite Logic Programs: Semantics

Definition 2.2: Rule derivation.
Let P a definite logic program and $R = (r_1, \ldots, r_n)$ be a sequence of rules in P such that

- each atom in the body of a rule r_i is a head of a rule r_j, where $j < i$.

$D_A = \{head(r) \mid r \in R\}$ is a derivation for an atom A on P iff $A \in D_A$.

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Definition 2.3: Closed and Ground sets of Atoms.
Let S be a set of atoms, P a definite program.

- S is closed under P iff $A \in S$ whenever $A \leftarrow B_1, \ldots, B_n \in P$ and $\{B_1, \ldots, B_n\} \subseteq S$.
- S is grounded in P iff $A \in S$ implies there is a derivation for A from P.

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We call S a consequence of P if it is closed and grounded in P, denoted by $Cn(P)$.

Definite logic Programs: Remarks

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Further logical remarks:

- each rule is a definite clause
  - definite clauses are disjunctions with exactly one positive atom:

$$a_0 \vee \neg a_1, \vee \ldots \vee \neg a_m$$

  - a set of definite clauses has a unique smallest model

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$$a_0 \vee \neg a_1, \vee \ldots \vee \neg a_m$$

  - a set of definite clauses has a unique smallest model
- horn clauses are clauses with at most one positive atom
  - every definite clause is a horn clause
  - a set of horn clauses has a unique smallest model or none

Definite Logic Programs

Example 2.4: A definite LP:

$a \leftarrow b.$

$b \leftarrow b.$

$c \leftarrow a.$

$c \leftarrow d.$

$d.$

$e \leftarrow a, b, c.$

$e \leftarrow c, d.$

Definite Logic Programs

Example 2.4: A definite LP:

a ←b.
b ←b.
c ←a.
c ←d.
d.
e ←a, b, c.
e ←c, d.

Example 2.5: Solution:

$$\{c, d, e\}$$

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So far, we only considered constant atoms, but how to handle variables?

How is it handled in predicate logic? $\rightarrow$ reminder on predicate logic

Interlude: Syntax of Terms in Predicate Logic

Definition 2.6: Let $\mathcal{F}$ be a set of function symbols (with arity), $\mathcal{V}$ a set of variable symbols. The set $\mathcal{T}_{\mathcal{F},\mathcal{V}}$ of **terms over $\mathcal{F}$ and $\mathcal{V}$** is the $\subset$-minimal set, such that:

- $\mathcal{V} \subseteq \mathcal{T}_{\mathcal{F},\mathcal{V}}$
- $t_1, \ldots, t_n \in \mathcal{T}_{\mathcal{F},\mathcal{V}}$ and $f/n \in \mathcal{F}$ implies $f(t_1, \ldots, t_n) \in \mathcal{T}_{\mathcal{F},\mathcal{V}}$.

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Example 2.7: $\mathcal{F} = \{1, 0, e, \pi, +, -\}$, $\mathcal{V} = \{X, Y, Z\}$

- We write $+/2 \in \mathcal{F}$ and $-/2 \in \mathcal{F}$
- Terms: $+(1, 0) = 1 + 0$, $-(\pi, +(e, 1)) = \pi - (e + 1)$, $+(X, -(\pi, Y)) = X + (\pi - Y)$
- no Terms: $\cdot(\pi, X) = \pi \cdot X$, $+(2, 2) = 2 + 2$, $-(\pi, 0, e)$

Interlude: Syntax of Atoms in Predicate Logic

Definition 2.8: Let $\mathcal{F}$ be a set of function symbols. The set $\mathcal{T}$ of **variable-free terms of** $\mathcal{F}$ is the $\subset$-minimal set, such that:
$t_1, \ldots, t_n \in \mathcal{T}$ and $f/n \in \mathcal{F}$ implies $f(t_1, \ldots, t_n) \in \mathcal{T}$.

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Example 2.9: $\mathcal{F} = \{ a/0, f/1 \} \rightsquigarrow \mathcal{T} = \{ a, f(a), f(f(a)), \ldots \}$; $\mathcal{F} = \{ g/1 \} \rightsquigarrow \mathcal{T} = \emptyset$.

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Definition 2.10: Let $\mathcal{P}$ a set of predicate symbols, $\mathcal{F}$ a set of function symbols, $\mathcal{V}$ a set of variable symbols. The set $\mathcal{A}$ of **atoms over $\mathcal{P}$, $\mathcal{F}$, and $\mathcal{V}$** is the $\subseteq$-minimal set, such that:
$t_1, \ldots, t_n \in \mathcal{T}_{\mathcal{F},\mathcal{V}}$ and $p/n \in \mathcal{P}$ implies $p(t_1, \ldots, t_n) \in \mathcal{A}$.

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Example 2.11: $\mathcal{P} = \{\text{even}/1, \leq/2, p/3\}$, $\mathcal{F} = \{1, 0, e, \pi, +/2, -/2\}$, $\mathcal{V} = \{X, Y, Z\}$.

- Atoms: $\text{even}(0)$, $\text{even}(1)$, $\leq(X, \pi)$, $p(1, +(1, Y), Y)$
- no Atoms: $\text{odd}(1)$, $\leq(1, 1, 1)$, $\text{even}(2)$

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Solution:

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How to check rules with variables against definite logic programs?

Variables - How to Handle Them?

Substitute the variables by terms.

A substitution to variable-free terms is called a ground-instantiation.

Two options:

- Construct an exhaustive derivation and find one matching substitution for each rule
- Create a ground instantiation of all variables in all rules . . .
  then solve the variable free set of rules as before

Derivation + Local Match

The basic concept of **Datalog**

- find a homomorphism to map variables in rule to be an applicable rule with ground atoms
- apply rules as long as possible
  semi-naive evaluation
- distinction between extensional and intentional database

Ground + Solve

The basic concept of **Answer Set Programming**[1]

[1] Example taken from Torsten Schaubs teaching slides on “Answer Set Solving in Practice”

Ground + Solve

The basic concept of **Answer Set Programming**[1]

Example 2.12: $P = \{\, r(a,b) \leftarrow, r(b,c) \leftarrow, s(X,Y) \leftarrow r(X,Y) \,\}$

$T = \{\, a, b, c \,\}$

$$A = \left\{ \begin{array}{l} r(a,a), r(a,b), r(a,c), r(b,a), r(b,b), r(b,c), r(c,a), r(c,b), r(c,c) \\ s(a,a), s(a,b), s(a,c), s(b,a), s(b,b), s(b,c), s(c,a), s(c,b), s(c,c) \end{array} \right\}$$

$g(P) =$

$\{r(a,b) \leftarrow, r(b,c) \leftarrow$

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[1] Example taken from Torsten Schaubs teaching slides on “Answer Set Solving in Practice”

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+ default negation

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Normal Logic Programs: Syntax

Definition 2.13: Syntax of rules:

$$A \leftarrow B_1, \ldots, B_n, \text{not } C_1, \ldots, \text{not } C_m$$

where A, the B_i and the C_j are ground atoms.

Normal Logic Programs: Answer Set Semantics

Definition 2.14: Let S be a set of atoms, P a normal program.

- S is closed under P iff $A \in S$ whenever
  $A \leftarrow B_1, \ldots, B_n, \text{not } C_1, \ldots, \text{not } C_m \in P$ and
  $B_1, \ldots, B_n \in S$, $C_1, \ldots, C_m \notin S$.
- S is grounded in P iff $A \in S$ implies there is a valid derivation for A from P.

An Answer Set (AS) of P is called a stable model if it is closed and grounded in P. We call $SM(P)$ the set of stable models of P.

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Definition 2.15: Valid derivation

- S defeats $Q \leftarrow B_1, \ldots, B_n, \text{not } C_1, \ldots, \text{not } C_m$ iff $C_j \in S, j \in \{1, \ldots, m\}$
- derivation is valid in S iff it is only based on rules undefeated by S.

Stable Model Semantics: Gelfond-Lifschitz Reduct

Definition 2.16: Let P be a (ground) normal program, S a set of atoms. P^S is the program obtained from P by

- eliminating all rules containing not C where $C \in S$,
- eliminating all negated literals from the remaining rules.

S is an answer set under stable model semantics for P iff $S = Cn(P^S)$.

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Remarks:

- P^S contains no default-negation
- therefore P^S is a definite logic program
- $Cn(P^S)$ has one unique result
- P can have many (or no) stable models
- a sub(super)set of a stable model cannot be a stable model too

Some Examples on ASP-Programs

$$P_0 = \{a \leftarrow b, c \quad b \leftarrow a, \quad c \leftarrow\}$$

$$P_1 = \{a \leftarrow \text{not } b, \quad b \leftarrow \text{not } a\}$$

$$P_2 = \{b \leftarrow \text{not } c, \quad c \leftarrow \text{not } b\}$$

$$P_3 = \{a \leftarrow \text{not } b, \text{not } c, \quad b \leftarrow \text{not } a, \text{not } c, \quad c \leftarrow \text{not } a, \text{not } b\}$$

$$P_4 = \{a \leftarrow \text{not } b, \quad a \leftarrow \text{not } c, \quad b \leftarrow \text{not } a, \quad b \leftarrow \text{not } c, \quad c \leftarrow \text{not } a, \quad c \leftarrow \text{not } b\}$$

$$Q = \{z \leftarrow \text{not } z\}$$

$$R = \{z \leftarrow a, \text{not } z\}$$

$$S = \{z \leftarrow \text{not } a, \text{not } z\}$$

Further Extensions for Datalog and ASP

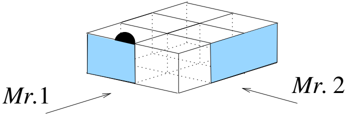
Datalog

- Tuple Generating Dependencies (existentially quantified variables - TGDs)
- Negation
- Constraints
- Various chase-variants with TGDs(e.g. skolem, restricted, core, ...)
- Various evaluations
- ...

Answer Set Programming

- Disjunctive heads
- Optimisation
- On-Demand Grounding
- Meta-ASP
- ...

Contextual Reasoning
Ghidini & Giunchiglias magic box



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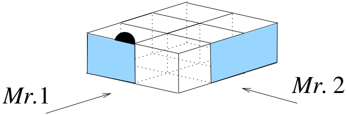

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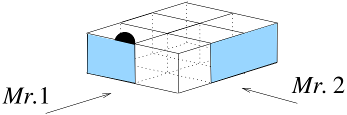

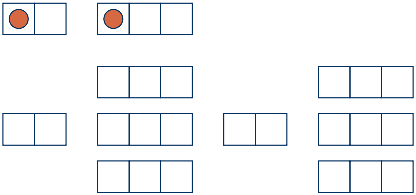

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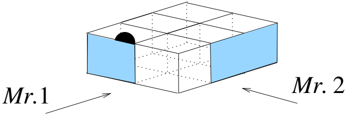

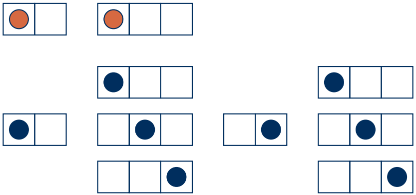

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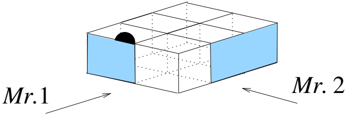

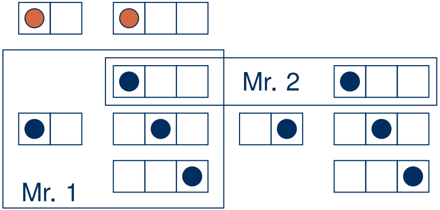

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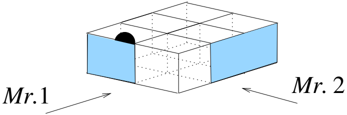

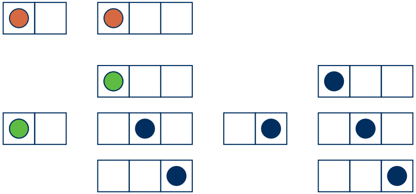

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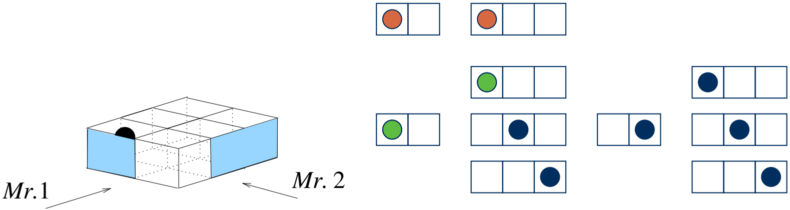

- model information
- integrate knowledge bases and context-based information
- synchronise knowledge, reasoning, and conclusions

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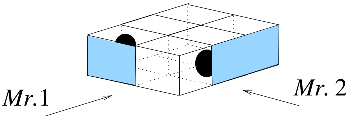

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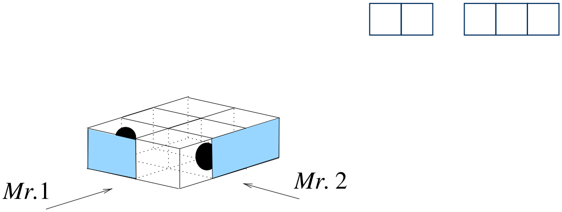

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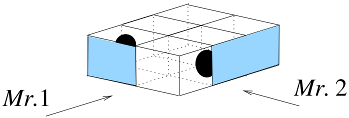
- synchronise knowledge, reasoning, and conclusions

Ghidini & Giunchiglias magic box



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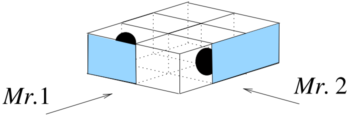
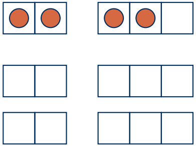
Contextual Reasoning

Ghidini & Giunchiglias magic box





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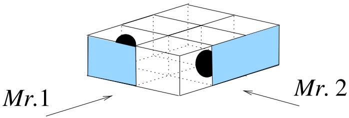

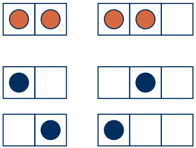

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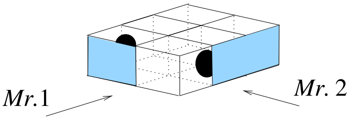

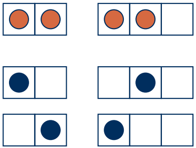

- model information
- integrate knowledge bases and context-based information
- synchronise knowledge, reasoning, and conclusions
- handle non-determinism and non-mononotonic behaviour

Possible Topics

- Answer Set Programming
- Datalog
- Distributed reasoning

Next Week . . .

Till next week . . .

have a look at the list and choose your favourite papers

Next Week . . .

- we will fix your topic to one paper
- we will discuss the format of the summary paper