Technische Universität Dresden

Prof. Dr. Sebastian Rudolph

Formal Concept Analysis

Exercise Sheet 7, Winter Semester 2015/16

Definition (closure system and closure operator).

(a) A set $\mathfrak{A} \subseteq \mathfrak{P}(M)$ *is a* closure system *on the set* M*, iff* $M \in \mathfrak{A}$ *and* $\mathfrak{X} \subseteq \mathfrak{A} \implies \bigcap \mathfrak{X} \in \mathfrak{A}$.

(b) A closure operator φ *on* M *is a map* φ *which maps each subset* $X \subseteq M$ *onto the corresponding* closure $\varphi(X) \subseteq M$ *such that*

1) $X \subseteq \varphi(X)$ *(extensive)*

2) $X \subseteq Y \implies \varphi(X) \subseteq \varphi(Y)$ *(monotone)*

3) $\varphi(\varphi(X)) = \varphi(X)$ *(idempotent)*

holds.

Exercise 1

Prove the following statements:

a) For any closure system $\mathfrak{A}$ on some set M, the mapping $\varphi_{\mathfrak{A}} : X \mapsto \bigcap_{X \subseteq Y \in \mathfrak{A}} Y$ is a closure operator on M.

b) For any closure operator φ on some set M, the family $\mathfrak{A}_\varphi := \{\varphi(X) \mid X \subseteq M\}$ is a closure system on M.

Exercise 2 (closure system)

Regard the "family context" $\mathbb{K}$:=({father, mother, daughter, son}, {old, young, male, female}, {(father, old), (father, male), (mother, old), (mother, female), (daughter, young), (daughter, female), (son, young), (son, male)}).

a) Explicitly list the elements of the map $\varphi\colon \mathfrak{P}(M) \to \mathfrak{P}(M)$ with $\varphi\colon B \mapsto B''$ and verify that φ is a closure operator.

b) Verify that the set of all concept intents of the family context is a closure system.

c) Draw a line diagram of the powerset of $\{\text{father}, \text{mother}, \text{daughter}, \text{son}\}$ and highlight the sets that have the same closure. Compare the diagram with the diagram of the concept lattice of the family context.

Exercise 3 (Next-Closure)

	old (1)	young (2)	male (3)	female (4)
father	×		×	
mother	×			×
son		×	×	
daughter		×		×

Compute all concept intents of the above "family context" using the Next-Closure algorithm. Compare your result with the concept intents from Exercise 2.

A	i	$(A\cap\{1,2,\ldots,i-1\})\cup\{i\}$ $A+i$	$(A+i)''$ $A\oplus i$	$A <_i A\oplus i$?	new intent