Intertranslatability of Labeling-Based Argumentation Semantics

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Abstract. Abstract Argumentation is a simple yet powerful formalism for modeling the human reasoning and argumentation process. Various semantics have been suggested with a view of arriving at coherent outcomes of the argumentation process. Two categories of semantics are well-known, extension-based semantics and labeling-based semantics. Translations between semantics are an important area of interest that enhance our understanding of the dynamics of various semantics and their structural and semantic interrelationship. The application of translations to extension-based semantics has been investigated in detail in the literature, however for labeling-based semantics which provide a more fine grained notion of acceptability such translations have not yet been studied. In this work, we fill this gab by investigating intertranslatability of labeling-based semantics. We show in which cases the existing results from the extension-based setting carry over to the labeling-based setting and we investigate intertranslatability between the three unique status semantics *grounded, ideal* and *eager*.

Keywords: Argumentation · Labeling-based semantics · Translations

1 Introduction

Argumentation theory and in particular abstract argumentation frameworks have become a popular field in artificial intelligence. In an abstract argumentation framework (AF) as introduced by Dung in 1995 [6], one can model scenarios with conflicting knowledge by considering only abstract entities called *arguments* and a binary relation between them the so-called *attack relation*. The inherent conflicts are solved on a semantical level usually by selecting sets of arguments, so-called *extensions* which can be accepted together. An alternative view on the semantics is in terms of labeling functions, where one assigns a label to each argument, depending on the specific semantics, denoting if it should be accepted (*in*), rejected (*out*) or undecided (*undec*) [5,13]. Thus, labeling-based semantics give a more fine grained notion of the status of each argument.

The notion of intertranslatability for the extension-based semantics has been investigated in much detail for most of the prominent semantics [8,9]. For two semantics σ, σ', intertranslatability involves translating an AF F to another AF

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F' through new arguments and new attacks between arguments such that the σ-extensions of F are in a certain relation to the σ'-extensions of F'. In case of extensions one just needs to compare the sets of accepted arguments, however when one considers labelings one needs to compare the status of each argument, as the transformation of the AF might also change the status of the *out* and *undec* labeled arguments.

Knowing about intertranslatability might become more and more important when it comes to the use of argumentation systems for the evaluation. In particular if one has an efficient system for semantics σ but one wants to evaluate an AF F w.r.t. semantics τ where no good implementations exits. Then, one would be interested in translating F into F' such that the σ-labelings of F are in a certain relation to the τ-labelings of F'.

The development of efficient systems to evaluate argumentation frameworks became a major topic. This is also reflected by the newly founded International Competition on Computational Models of Argumentation (ICCMA) which took place in 2015 for the first time [11]. Several argumentation systems use labeling-based algorithms in their computation [10,13], thus knowing about intertranslatability for labeling-based semantics can contribute to the development for such systems, or in the use of such systems.

The main contributions of this article are (i) the definition of *exact*, *faithful* and *weakly* translations for the labeling-based semantics, according to the intuition from [8,9]; (ii) we show under which conditions the results from the extension-based setting carry over to the labeling-based setting, in particular for the results on faithful translations we need to introduce an additional restriction on the translation to preserve the status of arguments labeled with *undec*; and (iii) we investigate intertranslatability between the unique-status semantics *grounded, ideal* and *eager* [3,7].

This article is organized as follows. In Sect. 2 we introduce the necessary background on abstract argumentation frameworks and the semantics in terms of extensions and labelings. In Sect. 3 we define the different types of translations for the labeling-based semantics, and in Sect. 4 we show which results from the extension-based setting carry over to the labeling-based one. Then, in Sect. 5 we analyze intertranslatability between the unique-status semantics grounded, ideal and eager. Finally, in Sect. 6 we conclude and discuss future directions.

2 Preliminaries

In this chapter we introduce argumentation frameworks. We then define various extension and labeling-based semantics. We also recall some results from other works which shall prove useful in our investigations.

Argumentation Frameworks were introduced by Dung [6]. Formally, an argumentation framework is a pair (A, R) where A is a set of arguments and $R \subseteq A \times A$ is the attack relation. The relation $(a, b) \in R$ means argument a attacks argument b. Similarly, a set of arguments $S \subseteq A$ attacks an argument $a \in A$ if and only if, $\exists b \in S$ such that $(b, a) \in R$.

Additionally, for a set $S \subseteq A$ of arguments, we denote by S^-, the set of all arguments that attack S, i.e., $S^- = \{b \mid \exists a \in S : (b, a) \in R\}$. For a set $S \subseteq A$ of arguments, we denote by S^+ the set of all arguments which are attacked by S, i.e., $S^+ = \{b \mid \exists a \in S : (a, b) \in R\}$. For $S \subseteq A$ and $a \in A$, we write $S \rightarrow a$, if there exists an argument $b \in S$ such that $(b, a) \in R$. Furthermore, an argument a is defended in an AF F by a set $S \subseteq A$ if for every $b \in A$, such that $(b, a) \in R$, $S \rightarrow b$. Lastly, the range of a set $S \subseteq A$, denoted by $S_R{}^+$, is defined as $S_R{}^+ = S \cup \{b \mid S \rightarrow b\}$. Argumentation frameworks can be represented as directed graphs with nodes representing arguments and edges representing attacks. We now define extension-based semantics drawing upon the works [1,2,6,12].

Let $F = (A_F, R_F)$ be an AF. A set $S \subseteq A$ is *conflict-free* in F, if there are no $a, b \in S$ such that $(a, b) \in R$. For a conflict-free set S:

- $S \in adm(F)$, if each $a \in S$ is defended by S;
- $S \in prf(F)$, if $S \in adm(F)$ and there is no $T \in adm(F)$ with $S \subset T$;
- $S \in com(F)$, if $S \in adm(F)$ and for each $a \in A$ that is defended by S, $a \in S$;
- $S \in grd(F)$, if $S \in com(F)$ and there is no $T \in com(F)$ with $T \subset S$;
- $S \in sem(F)$, if $S \in adm(F)$ and there is no $T \in adm(F)$ with $S_R{}^+ \subset T_R{}^+$;
- $S \in stb(F)$, if for each $a \in A \setminus S$, $S \rightarrow a$;
- $S \in stg(F)$, if there is no conflict-free set T in F, such that $T_R{}^+ \subset S_R{}^+$;
- $S \in idl(F)$, if $S \in adm(F)$ and S is the biggest set (w.r.t. set inclusion) such that for all $T \in prf(F)$, $S \subseteq T$;
- $S \in eag(F)$, if $S \in adm(F)$ and S is the biggest set (w.r.t. set inclusion) such that for all $T \in com(F)$, $S \subseteq T$.

Where *adm, prf, com, grd, sem, stb, stg, idl* and *eag* stand for admissible, preferred, complete, grounded, semi-stable, stable, stage, ideal and eager semantics.

Labeling-based semantics start by assigning a label from a set of labels $\Lambda = \{in, out, undec\}$ to every argument in an AF F. The set of labels, Λ, stands for accepted, rejected and undecided arguments respectively. The semantics then selects labelings from the set of all possible labelings which it sees as representing a coherent outcome of the conflicts in the AF. Another important notion is that of *'legally'* labeled.

- An *in*-labeled argument is said to be legally *in* if and only if all it's attackers are labeled *out*;
- An *out*-labeled argument is said to be legally *out* if and only if at least one of it's attackers is labeled *in*;
- An *undec*-labeled argument is said to be legally *undec* if and only if not all it's attackers are labeled *out* and it does not have an attacker that is labeled *in*.

In this work, we will denote by L, possibly indexed, a single labeling and $\mathcal{L}_\sigma(F)$ will represent the set of labelings for an AF F under a semantics σ.

We represent a labeling L for an AF F as a triple $L = (in(L), out(L), undec(L))$ where $in(L) = \{a \in A \mid L(a) = in\}$; $out(L) = \{a \in A \mid L(a) = out\}$; $undec(L) = \{a \in A \mid L(a) = undec\}$. For the set of *in*-labeled arguments of a labeling L, $in(L)$, we define $in(L)\downarrow_S$, the reduction of $in(L)$ to a set $S \subseteq A_F$ of arguments as: $in(L)\downarrow_S = \{in(L) \cap S\}$. $out(L)\downarrow_S$ and $undec(L)\downarrow_S$ are defined similarly. For a set of labelings of an AF F under the semantics σ, $\mathcal{L}_\sigma(F)$, the reduction of this set of labelings to a set of arguments S, $\mathcal{L}_\sigma(F)\downarrow_S$, is defined as: $\mathcal{L}_\sigma(F)\downarrow_S = \{(in(L) \cap S, out(L) \cap S, undec(L) \cap S) \mid L \in \mathcal{L}_\sigma(F)\}$.

Let L_1, L_2 be two labelings for an argumentation framework F. We say that L_2 is more or equally committed than $L_1 (L_1 \sqsubseteq L_2)$ iff $in(L_1) \subseteq in(L_2)$ and $out(L_1) \subseteq out(L_2)$. We can then characterize a labeling as being bigger or smaller than another labeling with respect to $\sqsubseteq$ which is a partial order.

We now introduce certain specific labeling-based semantics. A labeling L for an argumentation framework is said to be:

- **Admissible** if every *in*-labeled argument is legally in and every *out*-labeled argument is legally out.
- **Complete** if for all arguments $a \in A$: a is labeled *in* iff it is legally *in*; a is labeled *out* iff it is legally *out*; a is labeled *undec* iff it is legally *undecided*.
- **Grounded** if L is a complete labeling and $in(L)$ is minimal (w.r.t. set inclusion) among all complete labelings.
- **Preferred** if L is a complete labeling and $in(L)$ is maximal (with respect to set inclusion) among all complete labelings.
- **Semi-stable** if L is a complete labeling and $undec(L)$ is minimal (w.r.t set inclusion) among all complete labelings.
- **Stable** if it is a complete labeling with $undec(L) = \emptyset$.
- **Stage** if it is a conflict-free labeling where $undec(L)$ is minimal (w.r.t. set inclusion) among all conflict-free labelings.
- **Ideal** if it is the biggest admissible labeling (with respect to the partial order $\sqsubseteq$) that is smaller than or equal to each preferred labeling.
- **Eager** if it is the biggest admissible labeling (with respect to the partial order $\sqsubseteq$) that is smaller than or equal to each semi-stable labeling.

Among these semantics, *grounded*, *ideal* and *eager* labelings are unique status semantics in that they return a single, unique labeling for every AF. All other semantics are multiple status semantics which return possibly multiple labelings for every AF. *Stable* semantics is the only semantics that is not universally defined.

We now briefly recall some results from previous works which will help us in our investigations. From Caminada and Gabbay [5], we have that there is a bijective correspondence between complete extensions and complete labelings. It follows that for all completeness-based semantics, there is a bijective correspondence between the extension(s) and the labeling(s) for that semantics. All the semantics we consider in this work except *admissible* and *stage* semantics are completeness-based. We also recall from Caminada [4] that stage extensions and stage labelings are in a bijective correspondence. From Caminada [3], we have that the ideal and eager extensions (and hence the ideal and eager labelings) are also complete extensions (labelings). The proofs of these results are omitted here.

3 Translation Properties

By a translation, we mean an expansion of the source argumentation framework with further arguments and attacks, giving rise to the target argumentation framework. Formally, a translation Tr is defined as: $Tr = (A^*, R^*)$ where A^* is a set of additional arguments and R^* is the set of additional attack relations between arguments.

In this section, we first recall exactness and faithfulness properties of translations in the extension-based settings as defined in [9]. We then proceed to define exactness and faithfulness properties for translations in labeling-based semantics.

For two AFs $F = (A, R)$ and $F' = (A', R'), F \subseteq F'$ if and only if $A \subseteq A'$ and $R \subseteq R'$. A translation Tr is called *covering* if for every AF F, $F \subseteq Tr(F)$. A translation Tr is called *embedding* if for every AF F, $A_F \subseteq A_{Tr(F)}$ and $R_F = R_{Tr(F)} \cap (A_F \times A_F)$. We now recall the definitions of exactness and faithfulness properties of translations in the extension-based setting from [9]. For two extension-based semantics σ and σ', a translation Tr is called:

- **Exact**: if for every AF F, $\sigma(F) = \sigma'(Tr(F))$.
- **Weakly Exact**: if there exists S a given finite collection of (remainder) sets of arguments that are exclusively occurring in translated AFs, $\sigma(F) = \sigma'(Tr(F)) \setminus S$.
- **Faithful**: if for every AF F, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(Tr(F))\}$ and $|\sigma(F)| = |\sigma'(Tr(F))|$.
- **Weakly Faithful**: if there exists S a given finite collection of (remainder) sets of arguments that are exclusively occurring in translated AFs, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(Tr(F)) \setminus S\}$ and $|\sigma(F)| = |\sigma'(Tr(F)) \setminus S|$.

We now define *exactness* and *faithfulness* for labeling-based semantics. Intuitively, by exactness we mean that the labelings of the source AF under the semantics σ and those of the target framework under the semantics σ' coincide. Formally:

Definition 1. *A translation Tr is called* ***exact*** *for semantics $\sigma \Rightarrow \sigma'$ if for every AF F:*

1. $\forall L \in \mathcal{L}_\sigma(F)$: $\exists L' \in \mathcal{L}_{\sigma'}(Tr(F))$: $in(L) = in(L'), out(L) = out(L')\downarrow_{A_F}$, $undec(L) = undec(L')\downarrow_{A_F}$.
2. $|\mathcal{L}_\sigma(F)| = |\mathcal{L}_{\sigma'}(Tr(F))|$.

Definition 2. *A translation Tr is called* ***weakly exact*** *for semantics $\sigma \Rightarrow \sigma'$ if there exists a set of arguments A_p that are exclusively occurring in the translated AFs and a finite set of partial labelings $\mathcal{L}_p$ of A_p such that for every AF F and the remainder set $\mathcal{L}' = \{L \in \mathcal{L}_{\sigma'}(Tr(F)) \mid \exists L_p \in \mathcal{L}_p : L\downarrow_{(A_p \cap A_{Tr(F)})} = L_p\}$:*

1. $\forall L \in \mathcal{L}_\sigma(F) : \exists L' \in \mathcal{L}_{\sigma'}(Tr(F)) \setminus \mathcal{L}' : in(L) = in(L'), out(L) = out(L')\downarrow_{A_F}, undec(L) = undec(L')\downarrow_{A_F}$.
2. $|\mathcal{L}_\sigma(F)| = |\mathcal{L}_{\sigma'}(Tr(F)) \setminus \mathcal{L}'|$.

Intuitively, by faithful translations we mean translations that retain the original labelings of the source AF under the initial semantics. Formally:

Definition 3. *A translation Tr is called **faithful** for semantics $\sigma \Rightarrow \sigma'$ if for every AF F, $\mathcal{L}_\sigma(F) = \mathcal{L}_{\sigma'}(Tr(F))\downarrow_{A_F}$ and $|\mathcal{L}_\sigma(F)| = |\mathcal{L}_{\sigma'}(Tr(F))|$.*

Definition 4. *A translation Tr is called **weakly faithful** for semantics $\sigma \Rightarrow \sigma'$ if there exists a finite set of arguments A_p that are exclusively occurring in the translated AFs and a finite set of partial labelings $\mathcal{L}_p$ of labelings A_p such that for every AF F and the remainder set $\mathcal{L}' = \{L \in \mathcal{L}_{\sigma'}(Tr(F)) \mid \exists L_p \in \mathcal{L}_p : L\downarrow_{(A_p \cap A_{Tr(F)})} = L_p\}$: $\mathcal{L}_\sigma(F) = (\mathcal{L}_{\sigma'}(Tr(F)) \setminus \mathcal{L}') \downarrow_{A_F}$ and $|\mathcal{L}_\sigma(F)| = |\mathcal{L}'_\sigma(Tr(F)) \setminus \mathcal{L}'|$.*

Example 1. We now present an example to demonstrate the workings of a *weakly faithful* translation. Let $F = (\{a, b, c\}, \{(a, b), (b, c), (c, b)\})$ be an AF. The translation Tr_3 [9] is defined as: $Tr_3(F) = (A^*, R^*)$ where $A^* = A_F \cup \{t\}$ and $R^* = R_F \cup \{(a, t), (t, a) \mid a \in A_F\}$. The F target framework obtained from applying Tr_3 to F is depicted in Fig. 1.

We have that $\mathcal{L}_{stb}(F) = \{\{a, c\}, \{b\}, \emptyset\}$ and that $\mathcal{L}_{stg}(Tr_3(F)) = \{(\{a, c\}, \{b\}, \emptyset), (\{t\}, \{a, b, c\}, \emptyset)\}$. It is proven in [9] that Tr_3 is weakly exact for $stb \Rightarrow stg$ in the extension-based setting. By Theorem 2, we have that Tr_3 is embedding and weakly exact for $stb \Rightarrow stg$ in the labeling-based setting with $A_p = \{t\}$ and $\mathcal{L}' = \{(\{t\}, \{a, b, c\}, \emptyset)\}$.

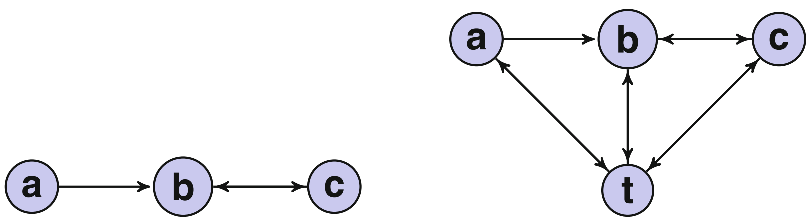

Fig. 1. The source AF F (left) and the target AF $Tr_3(F)$ (right)

4 Extension-Based and Labeling-Based Semantics Translation Comparison

Dvořák and Woltran [9] investigated intertranslatability between extension-based semantics and defined the notions of exactness and faithfulness for extension-based semantics. Having defined exactness and faithfulness for labeling-based semantics, in this section we investigate the relationship between the exactness and faithfulness of translations in extension-based setting to that in labeling-based setting.

First, we define a class of translations called *reserved translations* which will help simplify our investigations. We need the notion of reserved translations in order to be able to establish equivalence between faithfulness in extension-based and labeling-based setting since a translation which is faithful in extension-based setting maybe not be faithful in labeling-based because new arguments in the translation may attack arguments which were *undec* in the original framework and cause them to be *out* in the target framework. We say that a set of arguments in an AF F constitutes a cycle iff every argument in the set is reachable via the attack relation from every other argument in the set. The set of cycles of an AF is denoted by $cyc(F)$. The length of a cycle C is denoted by l_c. We define the function $\Psi(F)$ as:

$$\Psi(F) = \{C \in cyc(F) \mid \forall c \in A \setminus C, b \in C, (c, b) \in R : \{c\}^- \neq \emptyset\}$$

An argument $a \in A$ is *cycle-reachable* in F i.e. $a \in cr(F)$ iff one of the following conditions holds true:

1. $\exists C \in \Psi(F) : a \in C$
2. $\exists C \in \Psi(F)$ s.t. there exists a path from an argument $b \in C$ to a and no argument in the path is attacked by an argument which has no attackers.

The set of cycle-reachable arguments of an AF F is denoted by $cr(F)$.

Then *reserved translations* are translations where new arguments that attack cycle-reachable arguments in the original AF cannot be labeled *in* under any completeness-based semantics. For an AF F and a translation $Tr = (A^*, R^*)$ we define the function $\Omega(Tr(F))$ as:

$$\Omega(Tr(F)) = \{a \in A^* \mid \exists b \in cr(F) : (a, b) \in R^*\}$$

A translation Tr is called *reserved* iff one of the following conditions holds:

1. $\forall a \in \Omega(Tr(F)) : (a, a) \in R^*$
2. $\forall a \in \Omega(Tr(F)) : \exists c \in A' : (c, a) \in R^*, \{c\}^- = \emptyset$
3. $\forall a \in \Omega(Tr(F)) : a$ is cycle-reachable in $Tr(F)$.

Lemma 1. *Let $F = (A_F, R)$ be an AF and let σ be a completeness-based semantics. Then: $\forall a \in A_F : \exists L \in \mathcal{L}_\sigma(F) : L(a) = undec$ only if a is cycle-reachable in F.*

Proof. We do a proof by contradiction. Let $a \in A_F$ be an argument in F and for a labeling L under a completeness-based semantics σ, let $L(a) = undec$ and let a be non cycle-reachable.

Since $L(a) = undec$, by definition we have that there exists an argument $b \in A$ such that $(b, a) \in R$ and $L(b) = undec$. Now we have that either a attacks b or b has an attacker c and $L(c) = undec$. In the first case we get that (a, b) is a cycle and we have a contradiction. In the second case, we have that either b attacks c or c has an attacker d and $L(d) = undec$. Again, in the first case we have that (b, c) constitutes a cycle and we have a contradiction. In the second case, we have that either c attacks d or d has another undecided attacker. By the same token, we have that either there exists an infinite chain of undecided arguments or their exists an undecided argument x_i which is attacked by an undecided argument x_{i-1} which it also attacks. Since we confine ourselves to finite AFs, we have that (x_i, x_{i-1}) constitutes a cycle and hence that a is cycle-reachable which is a contradiction and this completes our proof. □

4.1 Exactness Comparison

We now derive the equivalences between translation properties in the extension-based and labeling-based settings.

Theorem 1. *Let σ, $\sigma' \in \{com, grd, prf, sem, stb, idl, eag\}$. A embedding translation Tr is exact for $\sigma \Rightarrow \sigma'$ in the extension-based setting, if and only if Tr is exact for $\sigma \Rightarrow \sigma'$ in the labeling-based setting.*

Proof. ⇒: Let a translation Tr be exact for $\sigma \Rightarrow \sigma'$ in the extension-based setting. Then, by definition, we have that for all AFs F, $\sigma(F) = \sigma'(Tr(F))$. Let $in(\mathcal{L}_\sigma(F))$ be the set of *in*-labeled arguments (extensions) of F under the semantics σ, i.e., $in(\mathcal{L}_\sigma(F)) = \{in(L) \mid L \in \mathcal{L}_\sigma(F)\}$. Let $in(\mathcal{L}_{\sigma'}(Tr(F))) = \{in(L) \mid L \in \mathcal{L}_{\sigma'}(Tr(F))\}$ be the same for the AF $Tr(F)$ and the semantics σ'. Since $\sigma(F) = \sigma'(Tr(F))$, we have that $in(\mathcal{L}_\sigma(F)) = in(\mathcal{L}_{\sigma'}(Tr(F)))$. Hence we have that $\forall L \in \mathcal{L}_\sigma(F) : \exists L' \in \mathcal{L}_{\sigma'}(Tr(F)) : in(L) = in(L')$. We note that since both σ, σ' are completeness-based and that it is proven in [5] that there is a bijective correspondence between complete extensions and complete labelings, we can conclude that

$$\forall L \in \mathcal{L}_\sigma(F) : \exists L' \in \mathcal{L}_{\sigma'}(Tr(F)) : (in(L) = in(L'), out(L) = out(L')\downarrow_{A_F}, \\ undec(L) = undec(L')\downarrow_{A_F}) \text{ and } |\mathcal{L}_\sigma(F))| = |\mathcal{L}_{\sigma'}(Tr(F))|$$

which completes our proof.

⇐: We know from Caminada and Gabbay [5] that there is a bijective correspondence between complete extensions and complete labelings and we have by definition that σ, σ' are completeness-based. Since $\sigma(F) = \{in(L) \mid L \in \mathcal{L}_\sigma(F)\}$ and $\sigma'(F) = \{in(L) \mid L \in \mathcal{L}_{\sigma'}(Tr(F))\}$ and since Tr is exact for $\sigma \Rightarrow \sigma'$ in the labeling-based setting, it follows that Tr is exact for $\sigma \Rightarrow \sigma'$ in the extension-based setting as well.

Theorem 2. *Let σ, $\sigma' \in \{com, grd, prf, sem, stb, idl, eag\}$. If an embedding translation Tr is weakly exact for $\sigma \Rightarrow \sigma'$ in the extension-based setting, then Tr is weakly exact for $\sigma \Rightarrow \sigma'$ in the labeling-based setting.*

Proof. Let Tr be a weakly exact translation in extension-based setting. By definition we have that there exists a set S of arguments (remainder sets) occurring exclusively in $Tr(F)$ such that $\sigma(F) = \sigma'(Tr(F)) \setminus S$. By the fact that there is a bijective correspondence between complete and stage extensions and stage and complete labelings we have that $|\mathcal{L}_\sigma(F)| = |\mathcal{L}_{\sigma'}(Tr(F)) \setminus \mathcal{L}'|$ where

$\mathcal{L}'$ is the set of labelings in $Tr(F)$ corresponding to the set of extensions S. By the fact that Tr is weakly exact in extension-based setting, we get that $in(\mathcal{L}_\sigma(F)) = in(\mathcal{L}_{\sigma'}(Tr(F)) \setminus \mathcal{L}')$. Since $in(\mathcal{L}_{\sigma'}(Tr(F)) \setminus \mathcal{L}')$ only contains original arguments from F and Tr is embedding (i.e. that no additional arguments between the original set of arguments are added) we get that $out(\mathcal{L}_\sigma(F)) = out(\mathcal{L}_{\sigma'}(Tr(F)) \setminus \mathcal{L}')\downarrow_{A_F}$ and that $undec(\mathcal{L}_\sigma(F)) = undec(\mathcal{L}_{\sigma'}(Tr(F)) \setminus \mathcal{L}')\downarrow_{A_F}$ which completes our proof. □

4.2 Faithfulness Comparison

Theorem 3. *Let σ, $\sigma' \in \{com, grd, prf, sem, stb, idl, eag\}$. If a reserved translation Tr is faithful for $\sigma \Rightarrow \sigma'$ in the extension-based setting then Tr is faithful for $\sigma \Rightarrow \sigma'$ in the labeling-based setting.*

Proof. Let a translation Tr be faithful for $\sigma \Rightarrow \sigma'$ in the extension-based setting. Then, by definition, we have that for all AFs F, $\sigma(F) = \sigma'(Tr(F))\downarrow A_F$ and $|\sigma(F)| = |\sigma'(Tr)|$. We note that since σ, σ' are both completeness-based and that it is proven in [5] that there is a bijective correspondence between complete extensions and complete labelings and between stage extensions and stage labelings, we get that $|\mathcal{L}_\sigma(F)| = |\mathcal{L}_{\sigma'}(Tr(F))|$. By definition of faithfulness in extension-based semantics, we have that $in(\mathcal{L}_\sigma(F)) = in(\mathcal{L}_{\sigma'}(Tr(F)))\downarrow_{A_F}$. By definition of a reserved translation we have that new arguments in the translation which attack cycle-reachable arguments in the original AF cannot be labeled *in* under any completeness-based semantics. In other words, we get that the new arguments added in $Tr(F)$ do not cause a potentially undecided argument in F to become *out* in $Tr(F)$. By definition we have that $out(\mathcal{L}_\sigma(F)) = \{x \in F \mid (a, x) \in R, a \in in(\mathcal{L}_\sigma(F))\}$ and $out(\mathcal{L}_{\sigma'}(Tr(F))) = \{x' \in Tr(F) \mid (a', x') \in R^*, a' \in in(\mathcal{L}_{\sigma'}(Tr(F)))\}$. Since $in(\mathcal{L}_\sigma(F)) = in(\mathcal{L}_{\sigma'}(Tr(F)))\downarrow_{A_F}$, we have that

$$out(\mathcal{L}_\sigma(F)) = out(\mathcal{L}_{\sigma'}(Tr(F)))\downarrow_{A_F}, undec(\mathcal{L}_\sigma(F)) = undec(\mathcal{L}_{\sigma'}(Tr(F)))\downarrow_{A_F}$$

which completes our proof. □

Theorem 4. *If a translation Tr is faithful for $\sigma \Rightarrow \sigma'$ in the labeling-based setting then Tr is faithful for $\sigma \Rightarrow \sigma'$ in the extension-based setting.*

Proof. Let a translation Tr be faithful for $\sigma \Rightarrow \sigma'$ in the labeling-based setting. Then, by definition, we have that:

$\mathcal{L}_\sigma(F) = \mathcal{L}_{\sigma'}(Tr(F)) \downarrow_{A_F}$ and $|\mathcal{L}_\sigma(F)| = |\mathcal{L}_{\sigma'}(Tr(F))|$. Reasoning from [5] and [4], we have that $\sigma(F) = in(\mathcal{L}_\sigma(F))$ and $\sigma'(Tr(F)) = in(\mathcal{L}_{\sigma'}(Tr(F)))$ and hence that $\mathcal{L}_{\sigma'}(Tr(F)) \downarrow_{A_F} = \sigma'(Tr(F)) \downarrow_{A_F}$. It follows that $\sigma(F) = \sigma'(Tr(F)) \downarrow_{A_F}$ and $|\sigma(F)| = |\sigma'(Tr(F)) \downarrow_{A_F}|$ which completes our proof. □

Theorem 5. *Let σ, $\sigma' \in \{com, grd, prf, sem, stb, idl, eag\}$. If an embedding and reserved translation Tr is weakly faithful for $\sigma \Rightarrow \sigma'$ in the extension-based setting, then Tr is weakly faithful for $\sigma \Rightarrow \sigma'$ in the labeling-based setting.*

Proof. Let a translation Tr be weakly faithful for $\sigma \Rightarrow \sigma'$ in the extension-based setting. Then, by definition we have that for all AFs F, there exists a set of extensions S such that $\sigma(F) = \sigma'(Tr(F)) \setminus S)\downarrow_{A_F}$ and that $|\sigma(F)| = |\sigma'(Tr) \setminus S|$. By the fact that there is a bijective correspondence between complete extensions and complete labelings and between stage extensions and stage labelings, we get that $|\mathcal{L}_\sigma(F)| = |\mathcal{L}_{\sigma'}(Tr(F)) \setminus \mathcal{L}'|$ where $\mathcal{L}'$ is the set of labelings corresponding to the extensions in S.

Since Tr is an embedded reserved translation, from the reasoning in proof of Theorem 3 and the fact that Tr is exact for $\sigma \Rightarrow \sigma'$ in the extension-based setting, we get that

$$in(\mathcal{L}_\sigma(F)) = in(\mathcal{L}_{\sigma'}(Tr(F)) \setminus \mathcal{L}')\downarrow_{A_F}, out(\mathcal{L}_\sigma(F)) = out(\mathcal{L}_{\sigma'}(Tr(F)) \setminus \mathcal{L}')\downarrow_{A_F}$$
$$undec(\mathcal{L}_\sigma(F)) = undec(\mathcal{L}_{\sigma'}(Tr(F)) \setminus \mathcal{L}')\downarrow_{A_F}$$

which completes our proof. □

4.3 Equivalence Theorem Results

Having established equivalences between translation properties in extension-based and labeling-based settings, we combine the equivalence theorems and the results about extension-based translations in [9] and in [8] to arrive at results about labeling-based translations. We present these results in table in Fig. 2. For example, we have from [9] that Tr_8 is exact for *grd* $\Rightarrow$ *prf* in the extension-based setting. By Theorem 1 we get that Tr_8 is exact for *grd* $\Rightarrow$ *prf* in the labeling-based setting as well. The naming and the numbering of translations follows the scheme used in the original works mentioned above. Translations 3.7, 3.8, 3.9 and 3.12 are from [8] and the rest are from [9].

	grd	*adm*	*stb*	*com*	*prf*	*sem*	*stg*
grd	id	$Tr_4 \cup Tr_8$/-	Tr_8/-	Tr_8/3.8	Tr_8/3.8	Tr_8/3.8	Tr_8/3.7
adm	-	id	-/-	Tr_1	-/-	-/-	-/-
stb	-	Tr_4	id	Tr_4	Tr_4	Tr_4	Tr_3
com	-	-/-	-/-	id	-	-/-	-/-
prf	-	-	-	-	id	Tr_1	3.9/-
sem	-	-	-	-	3.12	id	3.9/-
stg	-	-	-	-	-	Tr_2	id

Fig. 2. Summary of exact/faithful translations for labeling-based semantics obtained from equivalence theorems and results in [9] and [8]

5 Translations: Unique Status Semantics

We now introduce some translations related to the three unique status semantics whose intertranslatability has not been studied: *ideal*, *ground* and *eager*.

The first translation relates to *ideal* and *eager* semantics. For an AF $F = (A_F, R_F)$, Tr_1 is defined as: $Tr_1 = (A^*, R^*)$, where $A^* = A_F \cup \{a' \mid a \in A_F\}$ and $R^* = R_F \cup \{(a, a'), (a', a), (a', a') \mid a \in A_F\}$. It is proven in [9] that Tr_1 is an embedding and exact translation for *prf*⇒*sem* and *adm*⇒*com* in the extension-based setting.

Theorem 6. *The translation Tr_1 is exact for the semantics idl⇒eag.*

Proof. Recall the definition of exactness in labeling-based semantics from *Page 4*. Since both *ideal* and *eager* are unique status semantics by definition, i.e., that for every AF F both return one unique labeling. Hence we have that $|\mathcal{L}_{idl}(F)| = |\mathcal{L}_{eag}(Tr_1(F))| = 1$ and *Condition 2* is proven.

To prove *Condition 1*, let L be the ideal labeling of F and L' be the eager labeling of $Tr_1(F)$. Since all the additional arguments in Tr_1 are self-attacking, they do not appear *in*-labeled in any labeling of the AF $Tr_1(F)$. Hence $Tr_1(F)$ is essentially reduced to F. Let $in(L)$ be the set of *in*-labeled arguments of L and $in(L')$ be the same for L'. We have by definition [3] that: $in(L) \subseteq in(L')$. We now identify two cases:

1. $in(L) = in(L')$: Then we have that $out(L) = out(L') \cap A_F$ and that $undec(L) = undec(L') \cap A_F$ and hence, *Condition 1* is proven.
2. $in(L) \subset in(L')$: Assume $in(L) \subset in(L')$. Then there exists an argument $a \in A_F$ such that $a \in in(L) \subset in(L'))$ but $a \notin in(L)$. Since $a \in in(L')$, by the definition of *eager* semantics it follows that $a \in \bigcap_{i=1}^{i=n} in(L_i) : L_i \in \mathcal{L}_{sem}(Tr_1(F))$. Since the translation $Tr_1(F)$ is exact for *prf*⇒*sem*, it follows that

$$\bigcap_{i=1}^{i=n} in(L_i) : L_i \in \mathcal{L}_{sem}(Tr_1(F)) = \bigcap_{i=1}^{i=n} in(L_i) : L_i \in \mathcal{L}_{prf}(Tr_1(F))$$

   Hence we get that $a \in \bigcap_{i=1}^{i=n} in(L_i) : L_i \in \mathcal{L}_{prf}(Tr_1(F))$ and hence $a \in in(L)$, which is a contradiction to our assumption. Hence we get that $in(L) = in(L')$ and by the reasoning in *case 1* (above), we complete our proof. □

The next three results present negative results about translatability in unique status semantics.

Theorem 7. *There does not exist a covering, embedding and exact translation for eag⇒grd in the labeling-based setting.*

Proof. We do a proof by counter example. We provide an AF for which no covering, embedding and exact translation exists for *eag*⇒*grd*. Consider the AF $F = (A, R)$ where: $A = \{a, b\}$ and $R = \{(a, b), (b, a), (b, b)\}$.

Since we consider covering and embedding translations, we assume that the original attacks between the original arguments are retained and no additional attacks between them are added. Since $\mathcal{L}_{eag}(F) = (\{a\}, \{b\}, \emptyset)$, to prove that no exact translation exists it suffices to prove that for all covering and embedding translations Tr':

$$L_1 = (\emptyset, \emptyset, A^*) \notin \mathcal{L}_{com}(Tr'(F)) \longrightarrow L_2 = (\{a\}, \{b...\}, ...) \notin \mathcal{L}_{com}(Tr'(F))$$

This follows from that fact that if L_1 *is* a complete labeling of $Tr'(F)$, then by definition it is also the grounded labeling and our proof is complete. On the other hand, if L_1 *is not* a complete labeling of $Tr'(F)$, then we need to prove that a labeling of the form L_2 is not a complete labeling and hence cannot be a grounded labeling of $Tr'(F)$, which would complete our proof. Assume $L_2 = (\{a\}, \{b...\}, \{...\}) \in \mathcal{L}_{com}(Tr'(F))$. Since $L_2(a) = in$ and knowing that the translation is covering and embedding, we identify three cases:

1. the translation $Tr'(F)$ does not add any additional arguments that attack a. Since $Tr'(F)$ is covering and embedding, the original attack relations between a and b are retained. Since $in(L_2) = \{a\}$, we get that b does not have any *in*-labeled attackers. Since a and b have a mutual attack, we have that $L_1 = (\emptyset, \emptyset, A^*) \in \mathcal{L}_{com}(Tr'(F))$ which contradicts our assumption.
2. the translation $Tr'(F)$ adds additional arguments that attack a, but those arguments are labeled *out*. Then it follows that $\forall x \in a^-, \exists t \in A^*$ such that $(t, x) \in R^*$ and $L_2(t) = in$ and hence $in(L_2) \neq \{a\}$, which is a contradiction.
3. the translation $Tr'(F)$ adds additional arguments with mutual attacks to a, i.e., $\forall x \in a^-, (a, x) \in R^*$. Then it follows that $L_1 = (\emptyset, \emptyset, A^*) \in \mathcal{L}_{com}(Tr'(F))$ which contradicts our assumption. □

Theorem 8. *There does not exist a covering, embedding and exact translation for eag⇒idl semantics.*

Proof. We do a proof by counter example. We provide an AF for which no covering, embedding and exact translation exists for *eag⇒idl* in the labeling-based setting. Consider the AF $F = (A, R)$ where: $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (b, a), (b, c), (c, d), (d, e), (e, c)\}$ [3].

We have that $\mathcal{L}_{eag}(F) = (\{b, d\}, \{a, c, e\}, \emptyset)$ and that $in(\mathcal{L}_{eag}(F)) = \{b, d\}$. Since by definition we have that for every AF F $|\mathcal{L}_{eag}(F))| = |\mathcal{L}_{idl}(F))| = 1$, in order to prove that there does not exist a covering, embedding and exact translation of F for *eag⇒idl*, we need to prove that for all covering and embedding translations $Tr'(F)$: $in(\mathcal{L}_{eag}(F)) \neq in(\mathcal{L}_{idl}(Tr'(F)))$. It suffices to prove that for all covering and embedding translations $Tr'(F)$:

$$\exists L' \in \mathcal{L}_{idl}(Tr'(F)) \text{ s.t. } b \notin in(L') \text{ and } d \notin in(L')$$

Let $Tr'(F)$ be a covering and embedding translation and $L = (\{b, d\}, \{a, c, e...\}, \{...\}) \in \mathcal{L}_{idl}(Tr'(F))$. Then by definition of *Preferred* semantics, we have that there does not exist a labeling $L' \in \mathcal{L}_{prf}(Tr'(F))$ such that $in(L) \subseteq in(L')$. Since $L(\{a, c, e\}) = out$, the construction of F and the covering and embedding properties of $Tr'(F)$, we deduce that none of the additional arguments that attack the original arguments may be have been added by $Tr'(F)$ can be labeled *in* or *undec*. We now see that $L'' = (\{a\}, \{b, ..\}, \{...\})$ is a complete labeling of $Tr'(F)$ and since $in(L'') \not\subseteq in(L')$, L'' is a preferred labeling of $Tr'(F)$. As $in(L'') \cap in(L') \neq \{b, d\}$, we have that $L = (\{b, d\}, \{a, c, e...\}, \{...\})$ is not the ideal labeling of $Tr'(F)$, which completes our proof. □

Theorem 9. *There does not exist a covering, embedding and exact translation for idl⇒grd.*

Proof (Proof Sketch). We provide a proof sketch. We present the AF $F = (A, R)$ where: $A = \{a, b\}$ and $R = \{(a, b), (b, a), (b, b)\}$ as a counter-example. Since $\mathcal{L}_{idl}(F) = (\{a\}, \{b\}, \emptyset)$, by the same reasoning as in the previous proof we now need to prove that: for every translation $Tr' = (A^*, R^*)$:

$$L_1 = (\emptyset, \emptyset, A^*) \notin \mathcal{L}_{com}(Tr'(F)) \longrightarrow L_2 = (\{a\}, \{...\}, \{...\}) \notin \mathcal{L}_{com}(Tr'(F))$$

The truth of the premise presents two cases: (1) there is an argument $x \in Tr'(F)$ such that $(x, a) \in R^*$ and x does not have any attackers and (2) all arguments $c \in a^-$ are labeled *out*; both of which lead to the conclusion. □

The next result relates to translatability between *grounded* and the other two unique status semantics. We recall translation $Tr_{3.8}$ [8] as $Tr_{3.8} = (A^*, R^*)$ where:

$$A^* = A_F \cup \{\tilde{F}_i \mid F_i \subseteq F\}$$
$$R^* = R_F \cup \{(\tilde{F}_i, \tilde{F}_i), (\tilde{F}_i, a) \mid F_i \subseteq (A, R), a \in A_{F_i} \setminus in(\mathcal{L}_{grd}(F_i)\}$$

It is proven in [8] that $Tr_{3.8}$ is an embedding and exact translation for $grd \Rightarrow \{prf, com, sem\}$ in extension-based setting. The target AF obtained by applying $Tr_{3.8}$ to the AF $F = (\{a, b\}, \{a, b\})$ is depicted in Fig. 3.

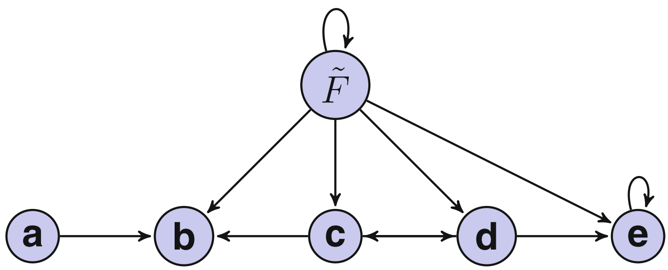

Fig. 3. The AF $Tr_{3.8}(F)$

Theorem 10. *The translation $Tr_{3.8}$ is exact for grd⇒{idl, eag}.*

Proof. We know from [9] that $Tr_{3.8}$ is exact for $grd \Rightarrow \{com, prf, sem\}$ in the extension-based setting. Since *grounded* is a unique status semantics, we have that, for every AF F, $|grd(F)| = |com(Tr_{3.8}(F))| = 1$ and $grd(F) =$

$com(Tr_{3.8}(F))$. By definition, we have that $com(Tr_{3.8}(F)) = prf(Tr_{3.8}(F)) = sem(Tr_{3.8}(F))$ and hence $com(Tr_{3.8}(F)) = idl(Tr_{3.8}(F)) = eag(Tr_{3.8}(F))$. We get that, for all AFs F, $grd(F) = idl(Tr_{3.8}(F)) = eag(Tr_{3.8}(F))$. Hence, we have that, for all AFs F,

$$in(\mathcal{L}_{grd}(F)) = in(\mathcal{L}_{idl}(Tr_{3.8}(F))) = in(\mathcal{L}_{eag}(Tr_{3.8}(F)))$$

and consequently that

$$out(\mathcal{L}_{grd}(F)) = out(\mathcal{L}_{idl}(Tr_{3.8}(F)))\downarrow_{A_F} = out(\mathcal{L}_{eag}(Tr_{3.8}(F)))\downarrow_{A_F}$$

and

$$undec(\mathcal{L}_{grd}(F)) = undec(\mathcal{L}_{idl}(Tr_{3.8}(F)))\downarrow_{A_F} = undec(\mathcal{L}_{eag}(Tr_{3.8}(F)))\downarrow_{A_F}$$

which completes our proof. □

Since *ideal*, *eager* and *grounded* are unique status semantics the notions of weakly exact and weakly faithful are not applicable to intertranslatability between these semantics.

6 Conclusion and Future Work

In this work, we built upon the investigations of Dvořák and Woltran [9] into the inter-translatability of extension-based semantics. We began our investigations by defining exactness and faithfulness of translations in the labeling-based setting. In order to establish faithfulness equivalence we defined a class of translations called reserved translations. We found that reserved translations which are exact or faithful in the extension-based setting are also exact or faithful in the labeling-based setting. This holds for all completeness based semantics. We also took into account the relatively new unique status semantics such as *ideal* and *eager*. We investigated and present results concerning the mutual inter-translatability of these three unique status semantics, *ideal*, *grounded* and *eager*.

There are promising directions for further research regarding translatability. One area of interest could be to examine the translatability of semantics in other classes of argumentation frameworks such as Abstract Dialectic Frameworks (ADF) especially the relationship between acceptance conditions of statements and AF semantics and translations between these semantics. Secondly, it would be interesting to explore translations between current semantics and various newly suggested semantics such as *cf2-semantics* and resolution based semantics in labeling-based setting.

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