Exercise 2.1. Let \( L_1, L_2 \) be two disjoint languages. Call \( L_1 \) and \( L_2 \) recursively inseparable if there does not exist a decidable language \( R \) such that \( L_1 \subseteq R \) and \( L_2 \cap R = \emptyset \). Show that the languages

\[
L_1 = \{ \langle M \rangle \mid M \text{ a Turing machine not accepting } \langle M \rangle \}, \\
L_2 = \{ \langle M \rangle \mid M \text{ a Turing machine accepting } \langle M \rangle \}.
\]

are recursively inseparable.

Exercise 2.2. Show that there cannot exist a reduction from \( A_{TM} \) to \( E_{TM} \).

Exercise 2.3. Let \( C \) be a property of Turing-recognizable languages. Show that if \( L_C \) is Turing-recognizable, then the finite languages in \( C \) are enumerable.

Exercise 2.4. Let

\[
T = \{ \langle M \rangle \mid M \text{ a TM that accepts } w^R \text{ whenever it accepts } w \},
\]

where \( w^R \) is the word \( w \) reversed. Show that \( T \) is undecidable. Is \( T \) Turing-recognizable?

Exercise 2.5. Show that every Turing-recognizable language can be mapping-reduced to \( HALT_{TM} \). In other words, show that \( HALT_{TM} \) is many-one complete (or \( m \)-complete) for the set of all Turing-recognizable languages.