Connecting Proof Theory and Knowledge Representation: Sequent Calculi and the Chase with Existential Rules

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Overview of Talk

I. Existential Rules and Chase Derivations

$$\forall \vec{x}\vec{y}\beta(\vec{x},\vec{y}) \rightarrow \exists \vec{z}\alpha(\vec{y},\vec{z})$$

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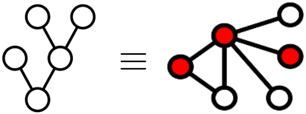
III. Correspondence



Existential Rules and Chase Derivations

Existential Rules 101: Basic Notions

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Database: Set $\{p_1(\vec{c}_1), \ldots, p_n(\vec{c}_n)\}$ of Ground Atoms

Example: $\mathcal{D} = \{male(Joe), human(Joe)\}$

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Existential Rule: $\forall \vec{x}\vec{y}\beta(\vec{x}, \vec{y}) \rightarrow \exists \vec{z}\alpha(\vec{y}, \vec{z})$

Example: $\rho = \forall x(male(x) \wedge human(x) \rightarrow \exists z(female(z) \wedge parent(z, x)))$

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Boolean Conjunctive Query (BCQ): $\mathcal{Q} = \exists \vec{x}(p_1(\vec{t}_1) \land \cdots \land p_n(\vec{t}_n))$

Example: $\mathcal{Q} = \exists z(female(z) \land parent(z, Joe))$

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BCQ Entailment Problem: Does $(\mathcal{D}, \mathcal{R}) \models \mathcal{Q}$ hold?

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Chase Derivation: $\mathcal{D}, (\rho_0, h_0, \mathcal{I}_1), \ldots, (\rho_n, h_n, \mathcal{I}_{n+1})$

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$\mathcal{D}$, $(\rho, h_1, \mathcal{I}_1)$, $(\rho, h_2, \mathcal{I}_2)$

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The Chase: Saturation of database under ERs

Sequent Calculi and Proofs

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Sequent: $\Gamma \vdash \Delta$ with Γ, Δ finite sets of FO formulae

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Terminology:

Γ is the *antecedent*

$\vdash$ is the *sequent arrow*

Δ is the *consequent*

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Interpretation: $\Gamma \vdash \Delta \quad \equiv \quad \bigwedge \Gamma \rightarrow \bigvee \Delta$

Example: $\mathtt{M}(a, b) \land \mathtt{M}(b, c) \rightarrow \exists x(\mathtt{A}(x, z) \land \mathtt{F}(x))$

Sequent Calculi 101: Sequent Calculus

$$\frac{}{\Gamma, p(\vec{t}) \vdash p(\vec{t}), \Delta}\,(id) \qquad \frac{\Gamma \vdash \phi, \Delta}{\Gamma, \neg\phi \vdash \Delta}\,(\neg_L) \qquad \frac{\Gamma, \phi \vdash \Delta}{\Gamma \vdash \neg\phi, \Delta}\,(\neg_R)$$

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Sequent Calculi 101: Existential Rules as Inference Rules

Corresponding Inference Rule:

$$\forall \vec{x}\vec{y}\beta(\vec{x},\vec{y}) \rightarrow \exists \vec{z}\alpha(\vec{y},\vec{z}) \quad \equiv \quad \frac{\Gamma, \beta(\vec{x},\vec{y}), \alpha(\vec{y},\vec{z}) \vdash \Delta}{\Gamma, \beta(\vec{x},\vec{y}) \vdash \Delta}\; s(\rho)\ \vec{z}\ \text{fresh}$$

Example: $\forall xy(\mathtt{M}(x,y) \rightarrow \exists z \mathtt{A}(z,x) \land \mathtt{F}(z)) \quad \equiv$

$$\frac{\Gamma, \mathtt{M}(x,y), \mathtt{A}(z,x), \mathtt{F}(z) \vdash \Delta}{\Gamma, \mathtt{M}(x,y) \vdash \Delta}\; s(\rho)\ z\ \text{fresh}$$

Correspondence

Querying via Sequent Derivations

Example: Does $(\mathcal{D}, \mathcal{R}) \models \exists x(\mathtt{A}(x, a) \wedge \mathtt{F}(x))$?

$\mathcal{D} = \{\mathtt{M}(b, a), \mathtt{M}(c, b)\}$

$\rho_1 = \forall xy(\mathtt{M}(x, y) \to \mathtt{A}(x, y) \wedge \mathtt{F}(x))$

$\rho_2 = \forall xy(\mathtt{A}(x, y) \wedge \mathtt{A}(y, z) \to \mathtt{A}(x, z))$

$$
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Sequent Derivations to Chase Derivations

Example: Does $(\mathcal{D}, \mathcal{R}) \models \exists x(\mathtt{A}(x,a) \land \mathtt{F}(x))$?

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \exists x(\mathtt{A}(x,a) \land \mathtt{F}(x)), \mathtt{A}(c,a)}\,(id) \qquad \dfrac{}{\Gamma \vdash \exists x(\mathtt{A}(x,a) \land \mathtt{F}(x)), \mathtt{F}(c)}\,(id)}{\Gamma \vdash \exists x(\mathtt{A}(x,a) \land \mathtt{F}(x)), \mathtt{A}(c,a) \land \mathtt{F}(c)}\,(\land_R)}{\mathtt{M}(b,a), \mathtt{A}(b,a), \mathtt{F}(b), \mathtt{M}(c,b), \mathtt{A}(c,b), \mathtt{F}(c), \mathtt{A}(c,a) \vdash \exists x(\mathtt{A}(x,a) \land \mathtt{F}(x))}\,(\exists_R)}{\mathtt{M}(b,a), \mathtt{A}(b,a), \mathtt{F}(b), \mathtt{M}(c,b), \mathtt{A}(c,b), \mathtt{F}(c) \vdash \exists x(\mathtt{A}(x,a) \land \mathtt{F}(x))}\,s(\rho_2)}{\mathtt{M}(b,a), \mathtt{A}(b,a), \mathtt{F}(b), \mathtt{M}(c,b) \vdash \exists x(\mathtt{A}(x,a) \land \mathtt{F}(x))}\,s(\rho_1)}{\mathtt{M}(b,a), \mathtt{M}(c,b) \vdash \exists x(\mathtt{A}(x,a) \land \mathtt{F}(x))}\,s(\rho_1)$$

1. $\mathcal{D} = \{\mathtt{M}(b,a), \mathtt{M}(c,b)\}$
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4. $\mathcal{I}_3 = \{\mathtt{M}(b,a), \mathtt{A}(b,a), \mathtt{F}(b), \mathtt{M}(c,b), \mathtt{A}(c,b), \mathtt{F}(c), \mathtt{A}(c,a)\}$

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$$

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Main Results and Future Work

Theorem

1. *A chase derivation* $(\rho_i, h_i, \mathcal{I}_i)_{i \in [n]}$ *witnessing* $(\mathcal{D}, \mathcal{R}) \models \mathcal{Q}$ *is transformable into a sequent proof of* $\mathcal{D} \vdash \mathcal{Q}$*, and vice-versa.*
2. *If* $(\mathcal{D}, \mathcal{R}) \not\models \mathcal{Q}$*, then proof search and the chase provide homomorphically equivalent counter-models of this fact.*

1. Sequent Proofs and Disjunctive Chase
2. Apply/Use Sequent Calculi Presented Here
   - 2.1 Combine with Other Proof Systems (e.g. Modal Reasoning)
   - 2.2 Explore Query-Decidable Classes of ERs