

# Knowledge Representation and Reasoning

## Horn Logics and Datalog Problems

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**Problem 1.** Consider the following set of formulae:

1.  $A \vee \neg A$
2.  $(\neg A \vee B) \wedge A \wedge (\neg C \vee A \vee B) \wedge \neg B$
3.  $(A \vee B) \wedge (A \vee \neg B \vee \neg C) \wedge (\neg A \vee B \vee \neg C)$

Decide which of these formulae are Horn-formulae.

**Problem 2.** Consider the following knowledge bases (1) in propositional logic and (2) in first-order logic:

**Knowledge Base (1):**

- $Cinema \vee SwimmingPool$
- $Cinema \rightarrow Fun$
- $SwimmingPool \vee Sleeping\_In \rightarrow Fun$

**Knowledge Base (2):**

- $\forall x GradStudent(x) \rightarrow Student(x)$
- $\forall x Student(x) \rightarrow HardWorker(x)$
- $GradStudent(Lisa)$

Use Resolution to:

1. Show  $KB_1 \models Fun$ .
2. Show  $KB_1 \not\models SwimmingPool$ .

3. Decide whether  $KB_2 \models \text{HardWorker}(\text{Lisa})$  holds.

**Problem 3.** Assume that  $\text{Parent}(x, y)$  is true iff (if and only if)  $x$  is a parent of  $y$ ,  $\text{Male}(x)$  is true iff  $x$  is male and  $\text{Female}(x)$  is true iff  $x$  is female. Write down Datalog rules that specify membership conditions for the binary familial relationship predicates  $\text{Father}$ ,  $\text{Mother}$ ,  $\text{Son}$ ,  $\text{Daughter}$ ,  $\text{Sibling}$ ,  $\text{Brother}$ ,  $\text{Sister}$ ,  $\text{Grandparent}$ ,  $\text{Uncle}$ , and  $\text{Cousin}$ . For example:

$$\text{Parent}(x, y) \wedge \text{Male}(x) \rightarrow \text{Father}(x, y)$$

Make the specification as precise as possible; e.g., in the case of  $\text{Father}$ , also add:

$$\begin{aligned} \text{Father}(x, y) &\rightarrow \text{Parent}(x, y) \\ \text{Father}(x, y) &\rightarrow \text{Male}(x) \end{aligned}$$

Try to be more succinct by reusing the specified predicates; e.g., use  $\text{Sibling}$  in the specification of  $\text{Brother}$  and  $\text{Sister}$ . Explain the nature of the problem in any case where expressivity limitations of Datalog make exact specification impossible.

**Problem 4.** Consider the Datalog knowledge base  $\mathcal{K} = \langle \mathcal{R}, \mathcal{F} \rangle$  where  $\mathcal{F}$  contains the following facts:

$$\mathcal{F} = \{\text{Father}(\text{john}, \text{mary}), \text{Mother}(\text{mary}, \text{peter}), \text{Father}(\text{john}, \text{david})\}$$

and  $\mathcal{R}$  contains the following rules:

$$\text{Parent}(x, y) \wedge \text{Parent}(y, z) \rightarrow \text{GrandParent}(x, z) \quad (1)$$

$$\text{Parent}(x, y) \wedge \text{Parent}(x, z) \rightarrow \text{Sibling}(y, z) \quad (2)$$

$$\text{Father}(x, y) \rightarrow \text{Parent}(x, y) \quad (3)$$

$$\text{Mother}(x, y) \rightarrow \text{Parent}(x, y) \quad (4)$$

Do the following:

1. Using backward-chaining show that John is a grand parent of Peter.
2. Using forward-chaining show that Mary and David are siblings.