Unsatisfiability of $G_{p,\top}$

Definition: $G_{p,\top}$ is obtained from G by deleting each clause, in which p occurs as a literal, and deleting each occurrence of $\neg p$.

Claim: If G is unsatisfiable, then $G_{p,\top}$ is unsatisfiable.

Proof. We proof the *contra-positive* statement:

If $G_{p,\top}$ is satisfiable, then G is satisfiable.

Assume that $G_{p,\top}$ is satisfiable. Then, there is a model I of $G_{p,\top}$. Without loss of generality, we can assume that $p^I = \top$, since the variable p does not occur in the formula $G_{p,\top}$ (see Problem 3.41). We now show that for every clause C appearing in G it holds that $C^I = \top$:
Let C be a clause appearing in G. If p appears as a literal in C, then $C^I = \top$ follows immediately from the definition of generalized disjunction. Otherwise, p does not appear as a literal in C. From the construction of $G_{p,\top}$, we know that there is a clause D appearing in $G_{p,\top}$ that is obtained from C by deleting each occurrence of $\neg p$. We know by assumption that $D^I = \top$, since I is a model of $G_{p,\top}$. Then, there is a literal L appearing in D with $L^I = \top$. We know that $L \neq \neg p$, since $\neg p^I = \bot$, and $L \neq p$ since p does not occur in D. Therefore, L appears in C as well. Then, $C^I = \top$. Consequently, I is a model of G. Hence, G is satisfiable.

Contraposition

Note that $(F \to G) \equiv (\neg F \vee G) \equiv (G \vee \neg F) \equiv (\neg\neg G \vee \neg F) \equiv (\neg G \to \neg F)$