



PROBLEM SOLVING AND SEARCH IN ARTIFICIAL INTELLIGENCE

Lecture 1

Sarah Gaggl

Dresden, 14th April 2015

What to expect

- The course has 12 lectures, 6 tutorials and a practical part
- Lecture Tuesdays in DS 3, 11:10-12:40
- Tutorials start in May and consist of exercise sheets
- Schedule and lecture material will be available at course web-page [https://ddl1.inf.tu-dresden.de/web/Problem_Solving_and_Search_in_Artificial_Intelligence_\(SS2015\)](https://ddl1.inf.tu-dresden.de/web/Problem_Solving_and_Search_in_Artificial_Intelligence_(SS2015))
- The practical part consists of solving (implementation) a problem and its presentation. Should be performed in groups of two, assignments will be ready at the end of May.

Literature

- Zbigniew Michalewicz and David B. Fogel. [How to Solve It: Modern Heuristics](#), volume 2. Springer, 2004.
- Stuart J. Russell and Peter Norvig. [Artificial Intelligence - A Modern Approach](#) (3. edition). Pearson Education, 2010.
- plus additional articles

Agenda

- 1 Introduction
- 2 Uninformed Search versus Informed Search (Best First Search, A* Search, Heuristics)
- 3 Local Search, Stochastic Hill Climbing, Simulated Annealing
- 4 Tabu Search
- 5 Answer-set Programming (ASP)
- 6 Constraint Satisfaction (CSP)
- 7 Structural Decomposition Techniques (Tree/Hypertree Decompositions)
- 8 Evolutionary Algorithms/ Genetic Algorithms

What are the Ages of my Three Sons?

Two men meet on the street. One gives the other a puzzle

A: "All three of my sons celebrate their birthday this very day! So, can you tell me how old each of them is?"

B: "Sure, but you'll have to tell me something about them."

A: "The product of the ages of my sons is 36."

B: "That's fine but I need more than just this."

A: "The sum of their ages is equal to the number of windows in that building."

B: "Still, I need an additional hint to solve your puzzle."

A: "My oldest son has blue eyes."

B: "Oh, this is sufficient!"

What are the Ages of my Three Sons? ctd.

"The product of the ages of my sons is 36."

son 1	son 2	son 3
36	1	1
18	2	1
12	3	1
9	4	1
9	2	2
6	6	1
6	3	2
4	3	3

What are the Ages of my Three Sons? ctd.

"The sum of their ages is equal to the number of windows in that building."

son 1	son 2	son 3
36	1	1
18	2	1
12	3	1
9	4	1
9	2	2
6	6	1
6	3	2
4	3	3

What are the Ages of my Three Sons? ctd.

"The sum of their ages is equal to the number of windows in that building."

$$\begin{array}{rcccccc} 36 & + & 1 & + & 1 & = & 38 \\ 18 & + & 2 & + & 1 & = & 21 \\ 12 & + & 3 & + & 1 & = & 16 \\ 9 & + & 4 & + & 1 & = & 14 \\ 9 & + & 2 & + & 2 & = & 13 \\ 6 & + & 6 & + & 1 & = & 13 \\ 6 & + & 3 & + & 2 & = & 11 \\ 4 & + & 3 & + & 3 & = & 10 \end{array}$$

What are the Ages of my Three Sons? ctd.

"The sum of their ages is equal to the number of windows in that building."

$$\begin{array}{r} 36 + 1 + 1 = 38 \\ 18 + 2 + 1 = 21 \\ 12 + 3 + 1 = 16 \\ 9 + 4 + 1 = 14 \\ \boxed{9 + 2 + 2 = 13} \\ \boxed{6 + 6 + 1 = 13} \\ 6 + 3 + 2 = 11 \\ 4 + 3 + 3 = 10 \end{array}$$

What are the Ages of my Three Sons? ctd.

"My oldest son has blue eyes."

$$36 + 1 + 1 = 38$$

$$18 + 2 + 1 = 21$$

$$12 + 3 + 1 = 16$$

$$9 + 4 + 1 = 15$$

$$9 + 2 + 2 = 13$$

$$6 + 6 + 1 = 13$$

$$6 + 3 + 2 = 11$$

$$4 + 3 + 3 = 10$$

What are the Ages of my Three Sons?

Two men meet on the street. One gives the other a puzzle

A: "All three of my sons celebrate their birthday this very day! So, can you tell me how old each of them is?"

B: "Sure, but you'll have to tell me something about them."

A: "The product of the ages of my sons is 36."

B: "That's fine but I need more than just this."

A: "The sum of their ages is equal to the number of windows in that building."

B: "Still, I need an additional hint to solve your puzzle."

A: "My oldest son has blue eyes."

B: "Oh, this is sufficient!"

What was difficult on this problem?

Problem Solving

- Where to begin?
- You have to create the **plan** for generating a solution.
- Always consider **all** of the **available data**.
- Can you make **connections** between the goal and what is given?

Why are Some Problems Difficult to Solve?

- The number of possible solutions in the **search space** is too large for an exhaustive search.
- The problem is too complicated, and simplified models of the problem are useless.
- The **evaluation function** of the quality of a solution is noisy or varies with time, which requires an entire series of solutions.
- There are so **many constraints** that finding even one feasible answer is difficult, let alone searching for an optimal solution.
- The person solving the problem is inadequately prepared.

The Size of the Search Space

Boolean Satisfiability Problem (SAT)

Make a compound statement of Boolean variables evaluate to **TRUE**.

- For example, consider the following problem of 100 variables given in conjunctive normal form (CNF):

$$F(x) = (x_{17} \vee \neg x_{37} \vee x_{73}) \wedge (\neg x_{11} \vee \neg x_{56}) \wedge \dots \wedge (x_2 \vee x_{43} \vee \neg x_{77} \vee \neg x_{89} \vee \neg x_{97}).$$

- **Challenge:** find the truth assignment for each variable x_i , for all $i = 1, \dots, 100$ s.t. $F(x) = \text{TRUE}$.

The Size of the Search Space

Boolean Satisfiability Problem (SAT)

Make a compound statement of Boolean variables evaluate to **TRUE**.

- For example, consider the following problem of 100 variables given in conjunctive normal form (CNF):

$$F(x) = (x_{17} \vee \neg x_{37} \vee x_{73}) \wedge (\neg x_{11} \vee \neg x_{56}) \wedge \cdots \wedge (x_2 \vee x_{43} \vee \neg x_{77} \vee \neg x_{89} \vee \neg x_{97}).$$

- **Challenge:** find the truth assignment for each variable x_i , for all $i = 1, \dots, 100$ s.t. $F(x) = \text{TRUE}$.

Space of possible solutions.

- Any binary string of length 100 is a possible solution.
- Two choices for each variable, and taken over 100 variables, generates 2^{100} possibilities.

The Size of the Search Space ctd.

- Size of the search space S is
 $|S| = 2^{100} \approx 10^{30} = 1\,000\,000\,000\,000\,000\,000\,000\,000\,000$.
- The number of bacterial cells on Earth is estimated at around 5×10^{30} .
- If we had a computer that could test 1000 strings per second and could have started at the beginning of time itself, 15 billion years ago (Big Bang!) we would have examined fewer than 1% of all the possibilities by now!
- Trying out all alternatives is out of the question.
- Choice of which evaluation function to use.
- Solutions closer to the right answer should yield better evaluations than those who are far away.
- If we try a string x and $F(x)$ returns TRUE, we are done. But what if $F(x)$ returns FALSE?
- How to find a function which gives more than just "right" or "wrong"?

The Size of the Search Space ctd.

Traveling Salesperson Problem (TSP)

- Given n cities and the distances between each pair of cities;
- Traveling salesperson must visit every city exactly once and return home covering the shortest distance.



The Size of the Search Space ctd.

Traveling Salesperson Problem (TSP)

- Given n cities and the distances between each pair of cities;
- Traveling salesperson must visit every city exactly once and return home covering the shortest distance.



Search Space

- Set of permutations of n cities.
- $2n$ different ways (for symmetrical TSP) to represent one tour.
- There are $n!$ ways to permute n numbers.
- $|\mathcal{S}| = n! / (2n) = (n - 1)! / 2$

The Size of the Search Space ctd.

- $|\mathcal{S}| = n!/(2n) = (n-1)!/2$
- For any $n > 6$, number of possible solutions to the TSP with n cities is larger than the number of possible solutions to the SAT problem with n variables.
- For $n = 6$: $5!/2 = 60$ solutions to the TSP and $2^6 = 64$ solutions to a SAT.
- For $n = 7$: 360 solutions to the TSP and 128 to the SAT.
- Search space increases very quickly with increasing n .
- A 50-city TSP has more solutions than existing liters of water on the planet.
- However, the evaluation function for the TSP is more straightforward than for SAT.
- Table with distances between each pair of cities.
- After n addition operations we could calculate the distance of any candidate tour and use this to evaluate its merit.
- $cost = dist(15, 3) + dist(3, 11) + \dots + dist(6, 15)$

Modeling the problem

- We only find the solution to a **model** of the problem.
- All models are simplifications of the real world.
- **Problem** \rightarrow **Model** \rightarrow **Solution**
 - 1 Use an approximate model of a problem and find the precise solution: **Problem** \rightarrow **Model_a** \rightarrow **Solution_p(Model_a)**
 - 2 Use a precise model of the problem and find an approximate solution: **Problem** \rightarrow **Model_p** \rightarrow **Solution_a(Model_p)**
- **Solution_a(Model_p)** is better than **Solution_p(Model_a)**.

Change over time

Problems may change

- before you model them,
- while you derive a solution, and
- after you execute the solution.

Traveling Salesperson:

- Travel time between two cities depends on many factors:
- traffic lights
- slow-moving trucks
- flat tire
- weather
- many more...

Constraints

- Almost all practical problems pose constraints
- Two types of constraints:
 - Hard constraints, and
 - Soft constraints.
- Constraints make the search space smaller, but
 - It is hard to create operators that will act on feasible solution and generate in turn new feasible solutions that are an improvement of previous solution.
 - The geometry of search space gets tricky.

Constraints ctd.

Timetable of the classes at a college in one semester

We are given

- list of **courses** that are offered;
- list of **students** assigned to each class;
- **professors** assigned to each class;
- list of available **classrooms**, and information for size and other facilities that each offer.

Constraints ctd.

Timetable of the classes at a college in one semester

We are given

- list of **courses** that are offered;
- list of **students** assigned to each class;
- **professors** assigned to each class;
- list of available **classrooms**, and information for size and other facilities that each offer.

Construct timetables that **fulfill hard constraints**:

- Each **class** must be assigned to an available **room** that has **enough seats** and requisite facilities.
- **Students** who are enrolled in **more than one class** can not have their classes held **at the same time** on the same day.
- Professors can not be assigned to teach courses that **overlap in time**.

Constraints ctd.

Timetable - Soft Constraints:

- Courses that meets *twice a week* should preferably be assigned to *Mondays and Wednesdays* or *Tuesdays and Thursdays*.
- Courses that meets *three times per week* should preferably be assigned to *Mondays, Wednesdays, and Fridays*.
- Course time should be assigned so that students do *not* have to take *final exams for multiple courses without any break in between*.
- If more than one room satisfies the requirements for a course and is available at the designated time, the course should be assigned to the room with the *capacity that is closest to the class size*.

Constraints ctd.

Timetable - Soft Constraints:

- Courses that meets **twice a week** should preferably be assigned to **Mondays and Wednesdays** or **Tuesdays and Thursdays**.
 - Courses that meets **three times per week** should preferably be assigned to **Mondays, Wednesdays, and Fridays**.
 - Course time should be assigned so that students do **not** have to take **final exams** for **multiple courses** without any **break in between**.
 - If more than one room satisfies the requirements for a course and is available at the designated time, the course should be assigned to the room with the **capacity that is closest to the class size**.
-
- Any timetable that **meets the hard constraints** is feasible.
 - The timetable has to be **optimized in the light of soft constraints**.
 - Each soft constraint has to be **quantified**.
 - We can **evaluate two candidate assignments** and decide that one is better than other.

Solve the Problem!

- Mr. Smith and his wife invited four other couples for a party.
- When everyone arrived, some of the people in the room shook hands with some of the others.
- Nobody shook hands with their spouse and nobody shook hands with the same person twice.
- After that, Mr. Smith asked everyone how many times they shook someone's hand.
- He received different answers from everybody.
- How many times did Mrs. Smith shake someone's hand?

Summary

Problem solving is difficult for several reasons:

- Complex problems often pose an enormous number of possible solutions.
- To get any sort of solution at all, we often have to introduce simplifications that make the problem tractable. As a result, the solutions that we generate may not be very valuable.
- The conditions of the problem change over time and might even involve other people who want to fail you.
- Real-world problems often have constraints that require special operations to generate feasible solutions.



References



Zbigniew Michalewicz and David B. Fogel.

How to Solve It: Modern Heuristics, volume 2. Springer, 2004.