Knowledge Representation and Reasoning
Uncertainty – Problems

Problem 1. *From the probability axioms we can deduce several further rules for computing probabilities. Consider the following ones:*

- *(R1)* $P(\neg A) = 1 - P(A)$.
- *(R2)* $P(A) = 0$, *if* A *is a contradiction.*

Deduce these rules from the probability axioms.

Problem 2. *Suppose that* $W = \{w_1, w_2, w_3\}$. *Define* m *as follows:*

- $m(w_1) = 1/4$
- $m(\{w_1, w_2\}) = 1/4$
- $m(\{w_2, w_3\}) = 1/2$
- $m(U) = 0$ *if* U *is not one of* $\{w_1\}, \{w_1, w_2\}, or \{w_2, w_3\}$

Provide all values for:

$Bel_m(w_1)$,$Bel_m(w_2)$, $Bel_m(w_3)$, $Bel_m(\{w_1, w_2\})$, $Bel_m(\{w_2, w_3\})$, $Bel_m(\{w_1, w_3\})$, $Bel_m(\{w_1, w_2, w_3\})$;

$Plaus_m(w_1)$,$Plaus_m(w_2)$, $Plaus_m(w_3)$, $Plaus_m(\{w_1, w_2\})$, $Plaus_m(\{w_2, w_3\})$, $Plaus_m(\{w_1, w_3\})$, $Plaus_m(\{w_1, w_2, w_3\})$.

Problem 3. *In a multi-sensor system, we need to combine evidences from multiple sensors. Assume that we have two sensors and three observable events with $W = \{A, B, C\}$. From the sensors, we get the following information that indicate the likelihood of observing each event:*

$$Sensor1 : m_1(A) = 0.95,\ m_1(B) = 0.05,\ m_1(C) = 0$$
$$Sensor2 : m_2(A) = 0.0,\ \ m_2(B) = 0.1,\ m_2(C) = 0.9$$

- Compute: $m_1 \oplus m_2(A)$, $m_1 \oplus m_2(B)$, $m_1 \oplus m_2(C)$
- Are the results satisfactory?

Problem 4. *The sorites paradox can be seen as one of the most striking paradoxes for logical reasoning (Hájek and Novák: 2003). Consider this paradox in the following form:*

(1) 10^{100} *is a huge number.*

(2) If n *is a huge number, then* $n - 1$ *is also huge.*

From this, deduce:

(3) 0 is a huge number.

Do the following:

- *Describe the problem here.*
- *State how fuzzy logic could help us, respecting the intuition that (2) seems not totally true, but almost true.*