Finite and algorithmic model theory – Exercises no. 3

In the following exercises you do not need to prove formally (e.g. by induction) that the duplicator has a winning strategy in your games. An idea how to play is sufficient. When looking for counterexamples, draw lines and circles.

Exercise 1

Show that a query checking if a given graph is two-colorable is not $\mathsf{FO}[\{E\}]$-definable.

Exercise 2

Show that a query checking if a given graph is acyclic is not $\mathsf{FO}[\{E\}]$-definable.

Exercise 3

Show that a query checking if a given graph is Eulerian is not $\mathsf{FO}[\{E\}]$-definable.

Exercise 4

Show that a query checking if a given graph is Eulerian is not $\mathsf{FO}[\{E\}]$-definable (use Kuratowski's theorem).

Exercise 5

During the lecture we have shown that if $\mathcal{L}_1, \mathcal{L}_2$ are linear orders of size $\geq 2^n$ then $\mathcal{L}_1 \equiv_n \mathcal{L}_2$. Prove that the exponential size of $\mathcal{L}_i$ is required for $\equiv_n$ to hold.

Below you can find some research ideas, related to EF-Games. I would be happy to work on it with an ambitious student (it should be good enough for a master thesis or research project).

- Read a recent paper by Fagin et al. LINK. It would be interesting to see whether his framework can be applied to other logics, e.g. fragments of FO or if this framework can be extended to work with counting quantifiers of the form $\exists^{\geq k}.\varphi$.