Exercise Sheet 2: First-Order Queries
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Exercise 2.1. Express the queries from Exercise 1.1 as domain independent FO-queries.

Exercise 2.2. Elaborate on why $r(x) \wedge \forall y.(q(x,y) \rightarrow p(x,y))[x]$ is domain independent.

Exercise 2.3. It was stated in the lecture that query mappings under the named perspective can be translated into query mappings under unnamed perspective. Specify this translation.

Exercise 2.4. Complete the proof that $\mathsf{RA}_{\mathrm{named}} \sqsubseteq \mathsf{DI}_{\mathrm{unnamed}}$ from the lecture by showing that the results of the transformation are (a) domain independent and (b) equivalent to the input query. In each case, show that the claimed property holds true for each case of the recursive construction under the assumption (induction hypothesis) that it has already been established for all subqueries.

Exercise 2.5. Complete the constructions for the proof of $\mathsf{AD} \sqsubseteq \mathsf{RA}$ given in the lecture.

1. Define the relational algebra expression $E_{a,\mathbf{adom}}$, such that $E_{a,\mathbf{adom}}(\mathcal{I}) = \{\{a \mapsto c\} \mid c \in \mathbf{adom}(\mathcal{I}, q)\}$ (assume that the query and the database schema are known).

2. Define the expressions E_φ for $\varphi = \varphi_1 \vee \varphi_2$ and $\varphi = \forall y.\psi$ in terms of expressions that have already been defined in the lecture.