TECHNISCHE UNIVERSITÄT DRESDEN

Artificial Intelligence, Computational Logic

ABSTRACT ARGUMENTATION

Introduction to Formal Argumentation

*slides adapted from Stefan Woltran's lecture on Abstract Argumentation

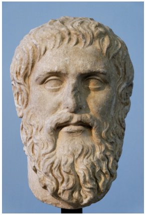
Sarah Gaggl

Dresden, 14th September 2015

DRESDEN concept Exzellenz aus Wissenschaft und Kultur

Outline

Argumentation in History



Plato's Dialectic

The dialectical method is discourse between two or more people holding different points of view about a subject, who wish to establish the truth of the matter guided by reasoned arguments.

The Republic (Plato), 348b

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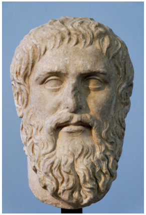

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Leibniz' Dream

"The only way to rectify our reasonings is to make them as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: Let us calculate [calculemus], without further ado, to see who is right."

Leibniz, Gottfried Wilhelm, The Art of Discovery 1685, Wiener 51

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Argumentation Nowadays

Abstract Argumentation [Dung, 1995]

- In abstract argumentation frameworks (AFs) statements (called arguments) are formulated together with a relation (attack) between them.
- Abstraction from the internal structure of the arguments.
- The conflicts between the arguments are resolved on the semantical level.

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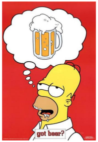

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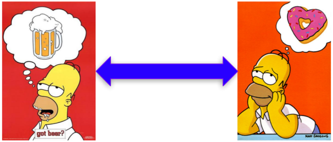

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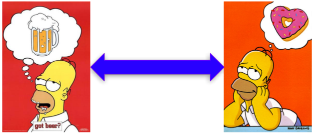

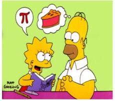

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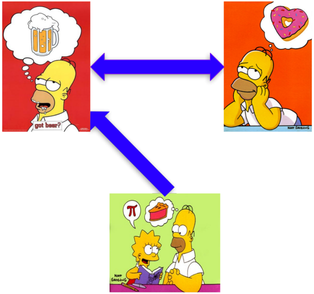

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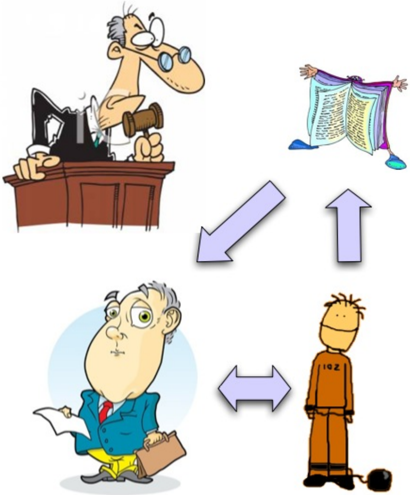
Legal Reasoning



Decision Support

Social Networks

Roadmap for the Lecture

Monday
- Introduction
- Abstract Argumentation Frameworks
- Semantics
- Computational Complexity

Tuesday
- Expressiveness of AFs
- Realizability
- Intertranslatability

Wednesday
- Notions of Equivalence

Thursday
- Argumentation and Answer-Set Programming (ASP)

Outline

Introduction

Argumentation:

. . . the study of processes "concerned with how assertions are proposed, discussed, and resolved in the context of issues upon which several diverging opinions may be held".

[Bench-Capon and Dunne, Argumentation in AI, AIJ 171:619-641, 2007]

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Formal Models of Argumentation are concerned with

- representation of an argument
- representation of the relationship between arguments
- solving conflicts between the arguments (“acceptability”)

Introduction (ctd.)

Increasingly important area

- “Argumentation” as keyword at all major AI conferences
- dedicated conference: COMMA, TAFA workshop; and several more workshops
- specialized journal: Argument and Computation (Taylor & Francis)
- two text books:
  - Besnard, Hunter: Elements of Argumentation. MIT Press, 2008
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Handbook of Formal Argumentation HOFA

- `http://formalargumentation.org`
- Volume 1 to appear in 2016

Introduction (ctd.)

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Applications

- PARMENIDES-system for E-Democracy: facilitates structured arguments over a proposed course of action [Atkinson et al.; 2006]
- IMPACT project: argumentation toolbox for supporting open, inclusive and transparent deliberations about public policy
- Decision support systems, etc.
- See also `http://comma2014.arg.dundee.ac.uk/demoprogram`.

Outline

The Overall Process

Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

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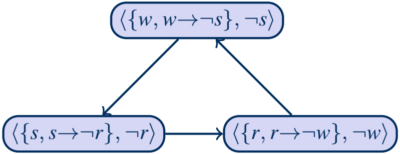

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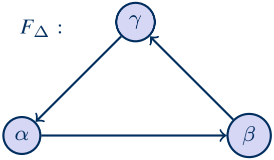

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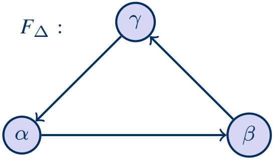

$$pref(F_\Delta) = \{\emptyset\}$$
$$stage(F_\Delta) = \{\{\alpha\}, \{\beta\}, \{\gamma\}\}$$

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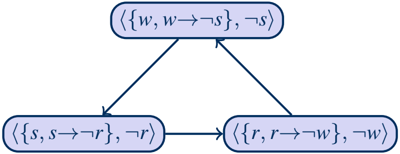

$$Cn_{pref}(F_\Delta) = Cn(\top)$$
$$Cn_{stage}(F_\Delta) = Cn(\neg r \vee \neg w \vee \neg s)$$

The Overall Process (ctd.)

Some Remarks

- Main idea dates back to Dung [1995]; has then been refined by several authors (Prakken, Gordon, Caminada, etc.)
- Separation between logical (forming arguments) and nonmonotonic reasoning ("abstract argumentation frameworks")
- Abstraction allows to compare several KR formalisms on a conceptual level ("calculus of conflict")

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Main Challenge

- All Steps in the argumentation process are, in general, intractable.
- This calls for:
  - careful complexity analysis (identification of tractable fragments)
  - re-use of established tools for implementations (reduction method)

Outline

Approaches to Form Arguments

Classical Arguments [Besnard & Hunter, 2001]

- Given is a KB (a set of propositions) Δ
- argument is a pair (Φ, α), such that $\Phi \subseteq \Delta$ is consistent, $\Phi \models \alpha$ and for no $\Psi \subset \Phi$, $\Psi \models \alpha$
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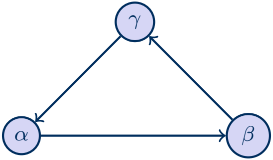
Other Approaches

- Arguments are trees of statements
- claims are obtained via strict and defeasible rules
- different notions of conflict: rebuttal, undercut, etc.

Outline

Dung's Abstract Argumentation Frameworks

Example



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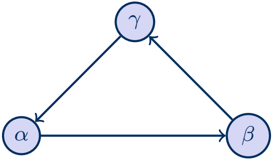

Main Properties

- Abstract from the concrete content of arguments but only consider the relation between them
- Semantics select subsets of arguments respecting certain criteria
- Simple, yet powerful, formalism
- Most active research area in the field of argumentation.
    - "plethora of semantics"

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Dung's Abstract Argumentation Frameworks

Definition

An argumentation framework (AF) is a pair (A, R) where

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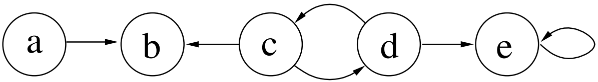
Example

$F = (\{a, b, c, d, e\}, \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\})$



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Conflict-Free Sets

Given an AF $F = (A, R)$.
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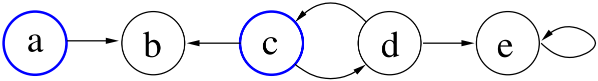

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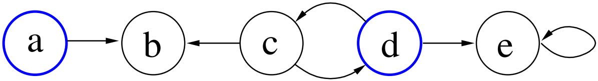

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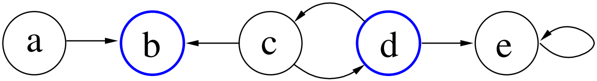

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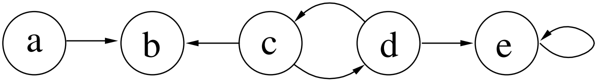

$cf(F) = \{\{a,c\},\{a,d\},\{b,d\},\{a\},\{b\},\{c\},\{d\},\emptyset\}$

Basic Properties (ctd.)

Admissible Sets [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in F, if

- S is conflict-free in F
- each $a \in S$ is defended by S in F
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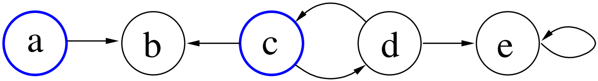

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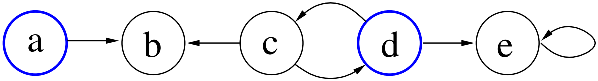

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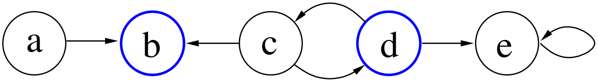

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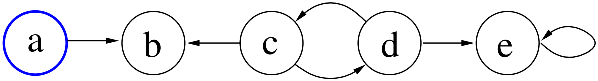

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Dung's Fundamental Lemma

Let S be admissible in an AF F and a, a' arguments in F defended by S in F. Then,

1. $S' = S \cup \{a\}$ is admissible in F
2. a' is defended by S' in F

Semantics

Naive Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a naive extension of F, if

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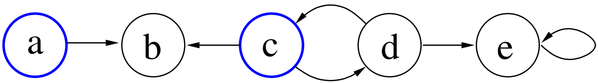

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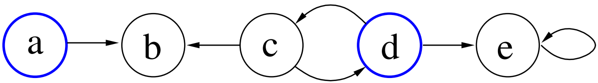

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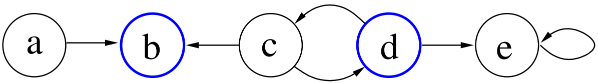

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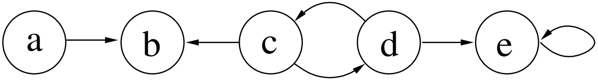

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Grounded Extension [Dung, 1995]

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1. put each argument $a \in A$ which is not attacked in F into S; if no such argument exists, return S;
2. remove from F all (new) arguments in S and all arguments attacked by them (together with all adjacent attacks); and continue with Step 1.

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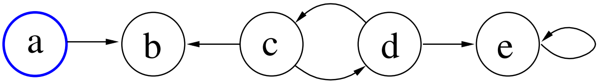
Example



$ground(F) = \{\{a\}\}$

Semantics (ctd.)

Complete Extension [Dung, 1995]

Given an AF (A, R). A set $S \subseteq A$ is complete in F, if

- S is admissible in F
- each $a \in A$ defended by S in F is contained in S
  - Recall: $a \in A$ is defended by S in F, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

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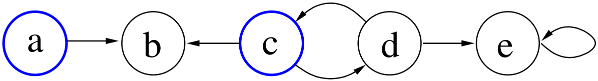

$comp(F) = \{\{a, c\},$

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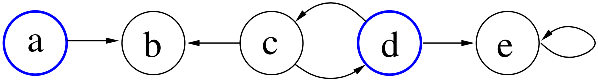

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Semantics (ctd.)

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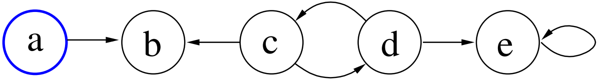

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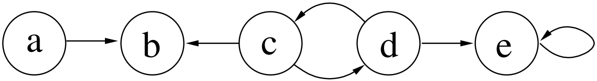

$comp(F) = \{\{a, c\}, \{a, d\}, \{a\}, \cancel{\{c\}, \{d\}, \emptyset}\}$

Semantics (ctd.)

Properties of the Grounded Extension

For any AF F, the grounded extension of F is the subset-minimal complete extension of F.

Semantics (ctd.)

Properties of the Grounded Extension

For any AF F, the grounded extension of F is the subset-minimal complete extension of F.

Remark

Since there exists exactly one grounded extension for each AF F, we often write $ground(F) = S$ instead of $ground(F) = \{S\}$.

Semantics (ctd.)

Preferred Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of F, if

- S is admissible in F
- for each $T \subseteq A$ admissible in F, $S \not\subset T$

Semantics (ctd.)

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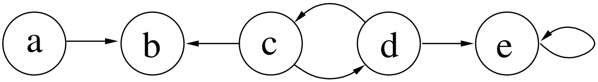

$pref(F) = \{\{a, c\}, \{a, d\}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$

Semantics (ctd.)

Stable Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stable extension of F, if

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- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$

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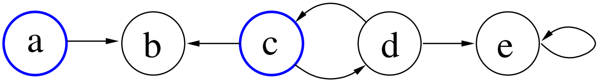

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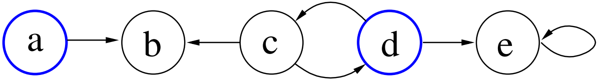

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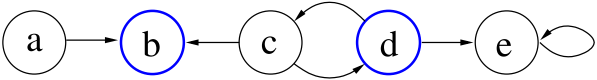

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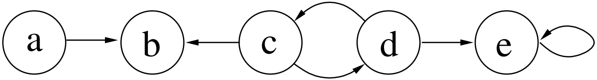

$stable(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{b, d\}}, \cancel{\{a\}}, \cancel{\{b\}}, \cancel{\{c\}}, \cancel{\{d\}}, \cancel{\emptyset}, \}$

Semantics (ctd.)

Some Relations

For any AF F the following relations hold:

1. Each stable extension of F is admissible in F
2. Each stable extension of F is also a preferred one
3. Each preferred extension of F is also a complete one

Semantics (ctd.)

Semi-Stable Extensions [Caminada, 2006]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a semi-stable extension of F, if

- S is admissible in F
- for each $T \subseteq A$ admissible in F, $S^+ \not\subset T^+$
  - for $S \subseteq A$, define $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$

Semantics (ctd.)

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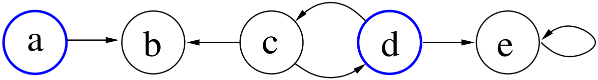

$semi(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{a\}}, \cancel{\{c\}}, \cancel{\{d\}}, \cancel{\emptyset}\}$

Semantics (ctd.)

Stage Extensions [Verheij, 1996]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stage extension of F, if

- S is conflict-free in F
- for each $T \subseteq A$ conflict-free in F, $S^+ \not\subset T^+$
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Ideal Extension [Dung, Mancarella & Toni 2007]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is an ideal extension of F, if

- S is admissible in F and contained in each preferred extension of F
- there is no $T \supset S$ admissible in F and contained in each of $\mathit{pref}(F)$

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Eager Extension [Caminada, 2007]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is an eager extension of F, if

- S is admissible in F and contained in each semi-stable extension of F
- there is no $T \supset S$ admissible in F and contained in each of $semi(F)$

Semantics (ctd.)

Properties of Ideal Extensions

For any AF F the following observations hold:

1. there exists exactly one ideal extension of F
2. the ideal extension of F is also a complete one

The same results hold for the eager extension and similar variants [Dvořák et al., 2011].

Semantics (ctd.)

Resolution-based grounded Extensions [Baroni,Giacomin 2008]

A resolution β of an AF $F = (A, R)$ contains exactly one of the attacks (a, b), (b, a) for each pair $a, b \in A$ with $\{(a, b), (b, a)\} \subseteq R$.

A set $S \subseteq A$ is a resolution-based grounded extension of F, if

- there exists a resolution β such that $\mathit{ground}((A, R \setminus \beta)) = S$
- and there is no resolution β' such that $\mathit{ground}((A, R \setminus \beta')) \subset S$

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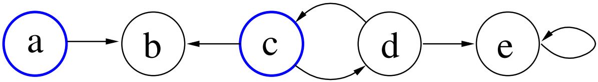

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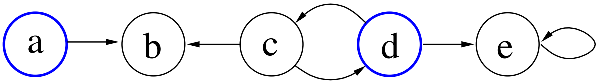


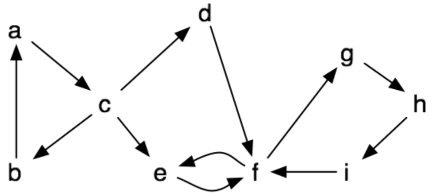
$ground^*(F) = \{\{a, c\}, \{a, d\}\}$

cf2 Semantics [Baroni, Giacomin & Guida 2005]

Definition (Separation)

An AF $F = (A, R)$ is called separated if for each $(a, b) \in R$, there exists a path from b to a. We define $[[F]] = \bigcup_{C \in SCCs(F)} F|_C$ and call $[[F]]$ the separation of F.

Example



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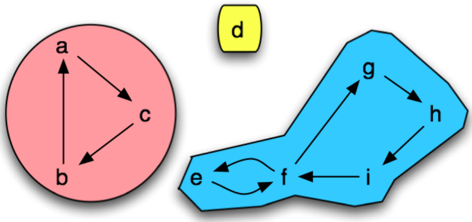

cf2 Semantics (ctd.)

Definition (Reachability)

Let $F = (A, R)$ be an AF, B a set of arguments, and $a, b \in A$. We say that b is reachable in F from a modulo B, in symbols $a \Rightarrow_F^B b$, if there exists a path from a to b in $F|_B$.

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Definition ($\Delta_{F,S}$)

For an AF $F = (A, R)$, $D \subseteq A$, and a set S of arguments,

$$\Delta_{F,S}(D) = \{a \in A \mid \exists b \in S : b \neq a, (b, a) \in R, a \not\Rightarrow_F^{A \setminus D} b\}.$$

By $\Delta_{F,S}$, we denote the lfp of $\Delta_{F,S}(\emptyset)$.

cf2 Semantics (ctd.)

cf2 Extensions [G & Woltran 2010]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a cf2-extension of F, if

- S is conflict-free in F
- and $S \in \mathit{naive}([[F - \Delta_{F,S}]])$.

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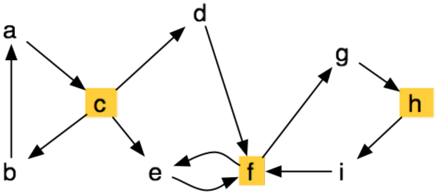
Example

$S = \{c, f, h\}$, $S \in cf(F)$.



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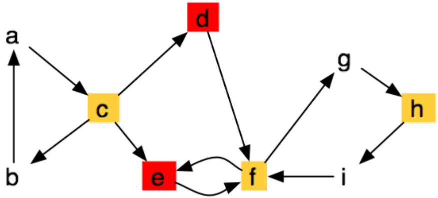
$S = \{c, f, h\},\ S \in cf(F),\ \Delta_{F,S}(\emptyset) = \{d, e\}$.



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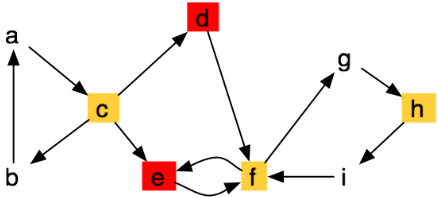

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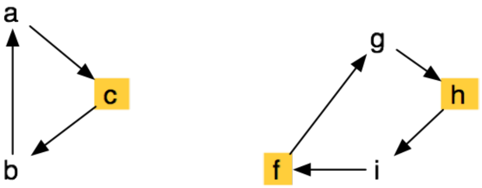
Example

$S = \{c, f, h\}$, $S \in cf(F)$, $\Delta_{F,S} = \{d, e\}$, $S \in naive([[F - \Delta_{F,S}]])$.



Outline

Relations between Semantics

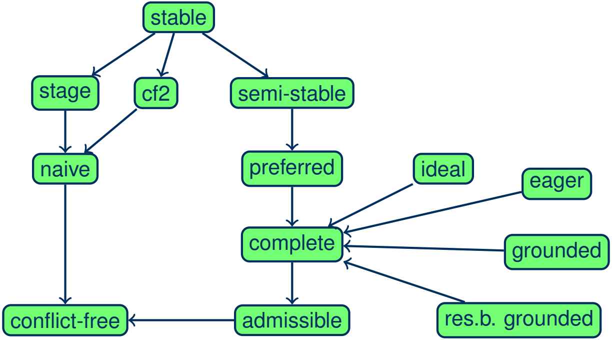


Figure : An arrow from semantics σ to semantics τ encodes that each σ-extension is also a τ-extension.

Outline

Decision Problems on AFs

Credulous Acceptance

Cred_σ: Given AF $F = (A, R)$ and $a \in A$; is a contained in at least one σ-extension of F?

Skeptical Acceptance

Skept_σ: Given AF $F = (A, R)$ and $a \in A$; is a contained in every σ-extension of F?

If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted[1].

[1] This is only relevant for stable semantics.

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Credulous Acceptance

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Hence we are also interested in the following problem:

Skeptically and Credulously accepted

Skept'_σ: Given AF $F = (A, R)$ and $a \in A$; is a contained in every and at least one σ-extension of F?

[1] This is only relevant for stable semantics.

Further Decision Problems

Verifying an extension

Ver_σ: Given AF $F = (A, R)$ and $S \subseteq A$; is S a σ-extension of F?

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Does there exist an extension?

Exists$_\sigma$: Given AF $F = (A, R)$; Does there exist a σ-extension for F?

Further Decision Problems

Verifying an extension

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Exists_σ: Given AF $F = (A, R)$; Does there exist a σ-extension for F?

Does there exist a nonempty extensions?

$\text{Exists}_\sigma^{\neg\emptyset}$: Does there exist a non-empty σ-extension for F?

Complexity Results (Summary)

Complexity for decision problems in AFs.

σ	Cred_σ	Skept_σ
ground	P-c	P-c
naive	in L	in L
stable	NP-c	co-NP-c
adm	NP-c	trivial
comp	NP-c	P-c
pref	NP-c	Π_2^p-c

σ	Cred_σ	Skept_σ
semi	Σ_2^p-c	Π_2^p-c
stage	Σ_2^p-c	Π_2^p-c
ideal	in Θ_2^p	in Θ_2^p
eager	Π_2^p-c	Π_2^p-c
*ground**	NP-c	co-NP-c
cf2	NP-c	co-NP-c

see [Baroni et al.2011, Coste-Marquis et al.2005, Dimopoulos and Torres1996, Dung1995, Dunne2008, Dunne and Bench-Capon2002, Dunne and Bench-Capon2004, Dunne and Caminada2008, Dvořák et al.2011, Dvořák and Woltran2010a, Dvořák and Woltran2010b]

Intractable problems in Abstract Argumentation

Most problems in Abstract Argumentation are computationally intractable, i.e. at least NP-hard. To show intractability for a specific reasoning problem we follow the schema given below:

Goal: Show that a reasoning problem is NP-hard.

Method: Reducing the NP-hard SAT problem to the reasoning problem.

- Consider an arbitrary CNF formula φ
- Give a reduction that maps φ to an Argumentation Framework F_φ containing an argument φ.
- Show that φ is satisfiable iff the argument φ is accepted.

Canonical Reduction

Definition

For $\varphi = \bigwedge_{i=1}^{m} l_{i1} \vee l_{i2} \vee l_{i3}$ over atoms Z, build $F_\varphi = (A_\varphi, R_\varphi)$ with

$$\begin{aligned}
A_\varphi &= Z \cup \bar{Z} \cup \{C_1, \ldots, C_m\} \cup \{\varphi\} \\
R_\varphi &= \{(z, \bar{z}), (\bar{z}, z) \mid z \in Z\} \cup \{(C_i, \varphi) \mid i \in \{1, \ldots, m\}\} \cup \\
&\quad \{(z, C_i) \mid i \in \{1, \ldots, m\}, z \in \{l_{i1}, l_{i2}, l_{i3}\}\} \cup \\
&\quad \{(\bar{z}, C_i) \mid i \in \{1, \ldots, m\}, \neg z \in \{l_{i1}, l_{i2}, l_{i3}\}\}
\end{aligned}$$

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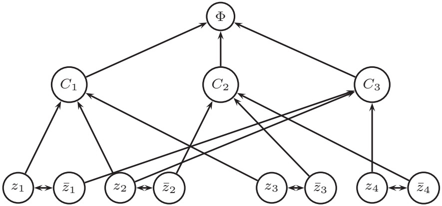
Example

Let $\Phi = (z_1 \vee z_2 \vee z_3) \wedge (\neg z_2 \vee \neg z_3 \vee \neg z_4) \wedge (\neg z_1 \vee z_2 \vee z_4)$.



Canonical Reduction: CNF $\Rightarrow$ AF (ctd.)

Theorem

The following statements are equivalent:

1. φ is satisfiable
2. F_φ has an admissible set containing φ
3. F_φ has a complete extension containing φ
4. F_φ has a preferred extension containing φ
5. F_φ has a stable extension containing φ

Complexity Results

Theorem

1. Cred_{stable} is NP-complete
2. Cred_{adm} is NP-complete
3. Cred_{comp} is NP-complete
4. Cred_{pref} is NP-complete

Proof.

(1) The hardness is immediate by the last theorem.
For the NP-membership we use the following guess & check algorithm:

- Guess a set $E \subseteq A$
- verify that E is stable
  - for each $a, b \in E$ check $(a, b) \notin R$
  - for each $a \in A \setminus E$ check if there exists $b \in E$ with $(b, a) \in R$

As this algorithm is in polynomial time we obtain NP-membership. $\square$

Outline

Argumentation Systems

Tools with web-interface

- ASPARTIX `http://rull.dbai.tuwien.ac.at:8080/ASPARTIX/`
- ConArg `http://www.dmi.unipg.it/conarg/`

Further Systems

- See first International Competition on Computational Models of Argumentation (ICCMA) `http://argumentationcompetition.org`

Outline

Exercises

1. Give an AF F such that $\mathit{stable}(F) = \emptyset$ and $\mathit{semi}(F) \neq \{\emptyset\}$.

2. Show that the following statement holds for any AF F.
   If $\mathit{stable}(F) \neq \emptyset$ then $\mathit{stable}(F) = \mathit{semi}(F) = \mathit{stage}(F)$.

3. Select three different semantics $\sigma, \sigma', \sigma''$ out of $\{\mathit{pref}, \mathit{ideal}, \mathit{semi}, \mathit{eager}, \mathit{ground}, \mathit{stable}\}$ of your choice and provide three pairs of AFs such that
   - $\sigma(F_1) = \sigma(G_1)$ but $\sigma'(F_1) \neq \sigma'(G_1)$
   - $\sigma'(F_2) = \sigma'(G_2)$ but $\sigma''(F_2) \neq \sigma''(G_2)$
   - $\sigma''(F_3) = \sigma''(G_3)$ but $\sigma(F_3) \neq \sigma(G_3)$

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