Exercise 5.1. Show that if $P = NP$, then a polynomial-time algorithm exists that produces a satisfying assignment of a given propositional formula.

Exercise 5.2. Let $\phi$ be a formula in CNF such that each clause in $\phi$ contains at most one literal that is negated. Denote with $CSAT_H$ the set of all such formulas that are in addition satisfiable. Show $CSAT_H \in P$.

* Exercise 5.3.

1. Show for an undirected graph $G$ and vertices $s$ and $t$ in $G$, that deciding whether there exists an undirected Hamiltonian path from $s$ to $t$ is NP-complete.

   **Hint:** Find a reduction from the corresponding problem on directed graphs.

2. Show that finding longest paths in undirected graphs, i.e.,

   \[ LPATH = \{ \langle G, s, t, k \rangle | G \text{ contains a simple path from } s \text{ to } t \text{ of length at least } k \} \]

   is NP-complete.

* Exercise 5.4. Denote with

   \[ CSAT = \{ \phi | \phi \text{ a satisfiable propositional formula in CNF} \} \]

   the satisfiability problem of propositional formulas in conjunctive normal form. Show that $CSAT$ is NP-complete by showing the for each propositional formula $\phi$ there exists an equi-satisfiable propositional formula $\phi'$ in CNF such that the length of $\phi'$ is polynomial in the length of $\phi$.

* Exercise 5.5. Show that the following problem is undecidable: given $k \in \mathbb{N}$ and some polynomial-time bounded Turing machine $M$, is the running time of $M$ on input of length $n$ in $O(n^k)$?