Knowledge Representation and Reasoning
Propositional Logic and FOL - Problems

Problem 1. *Consider the following propositional logic formulas α and β:*

$$\alpha = (\mathsf{Alarm\_Rings} \rightarrow \mathsf{Burglary}) \rightarrow (\mathsf{Alarm\_Rings} \vee \neg \mathsf{Burglary})$$
$$\beta = (\mathsf{Alarm\_Rings} \rightarrow \mathsf{Burglary}) \rightarrow (\mathsf{Alarm\_Rings} \wedge \mathsf{Dog\_Barks} \rightarrow \mathsf{Burglary})$$

Provide an answer to the following questions:

1. *Write down all subformulas of β.*
2. *How many interpretations are there for α? And for β?*
3. *Let $\mathcal{I}$ be the interpretation for α mapping* Alarm_Rings *to* true *and* Burglary *to* false*. Evaluate the formula to determine whether $\mathcal{I}$ is a model of α.*
4. *Let $\mathcal{I}'$ be the interpretation for β mapping all the propositional letters in β to* true*. Evaluate the formula to determine whether $\mathcal{I}'$ is a model of β.*
5. *Is α valid?*

Problem 2. *Suppose that we are given the following statements. If a tumor is Stage 4, then it exhibits metastasis. If it is not Stage 4, then it does not exhibit metastasis and it is a benign tumor. If a tumor is benign, then it has a good prognosis and if it has a good prognosis then it has a high survival rate. Finally, the tumor has high survival rate if and only if it does not have low survival rate.*

Formalise these statements using a knowledge base in the language of propositional logic. Does your formalisation entail that, if the tumor is Stage 4, then it has low survival rate? Does your formalisation entail that, if it is not Stage 4, then it has a good prognosis?

Problem 3. *Logic-based knowledge representation formalisms are currently being used in several countries to describe electronic patient records (e.g., by specifying knowledge about human anatomy, drugs, surgical procedures, and so on). Suppose that you have been hired by a medical research company to write a FOL knowledge base about congenital heart conditions (to be used in the annotation of patient data) from the informal specification given next:*

1. *Ventricular septum defect and aortic stenosis are kinds of congenital heart defects.*

2. *A ventricular septum defect affects the septum.*

3. *The septum is a tissue that is a part of both the left and the right hand sides of the heart.*

4. *The heart is an organ.*

5. *GATA4 is a kind of gene that is related to ventricular septal defects.*

6. *Every heart with a displaced tricuspid valve suffers from a congenital heart defect.*

Write down a FOL knowledge base $\mathcal{K}$ conforming to these specifications. Use the following unary and binary predicates to write $\mathcal{K}$:

- *Unary predicates:* VSD *(ventricular septum defect),* AS *(aortic stenosis),* CHD *(congenital heart defect),* Septum, Tissue, LHeart *(left hand side of the heart),* Heart, RHeart *(right hand side of the heart),* Organ, GATA4, Gene, TricValve *(tricuspid valve).*

- *Binary predicates:* affects, partOf, relatedTo, suffersCondition, hasDisPart.