Sarah Gaggl<br>LPArg Group

## Scalable Understanding

Navigation Approaches for Answer Sets
online, 20th October 2022

## Motivation

## Searching for the Right Bicycle



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## Combinatorial Search Problems



Figure 1: Bunt Vektor erstellt von macrovector - de.freepik.com

## Answer Set Programming (ASP)



## Declarative problem solving

- planning
- product configuration



## The NAVAS Project

## Navigation Approaches for Answer Sets

BMBF fundet project 10/2020-09/2024

## Goals

Allow for an interactive navigation within the ASP solution space by

1. Development of methods to navigate within the ASP solution space
2. Prototypical implementation of those methods with efficient algorithms
3. Evaluation on use cases configuration and argumentation

Team:


## Outline

- Preliminaries
- Weighted Faceted Navigation
- Incremental Answer Set Counting
- Diverse Answer Sets
- Visual Approach for Solution Space Exploration
- Conclusion


## Preliminaries

## Definition (logic program)

A (normal disjunctive) logic program $\Pi$ over a set of atoms $\left\{\alpha_{0}, \ldots, \alpha_{\mathrm{n}}\right\}$ is a finite set of rules $r$ of the form:

$$
\alpha_{0}|\ldots| \alpha_{\mathrm{k}} \leftarrow \alpha_{\mathrm{k}+1}, \ldots, \alpha_{\mathrm{m}}, \sim \alpha_{\mathrm{m}+1}, \ldots, \sim \alpha_{\mathrm{n}} \text {. where } 0 \leq \mathrm{k} \leq \mathrm{m} \leq \mathrm{n}
$$

Remark: We focus on ground programs without extended rules.

$$
\begin{gathered}
\mathcal{A S}(\Pi) \ldots \text { answer sets (solutions) } \\
2^{\mathcal{A} \mathcal{S}(\Pi)} \ldots \text { solution space } \\
\mathcal{B C}(\Pi):=\bigcup \mathcal{A S}(\Pi) \ldots \text { brave consequences } \\
\alpha \in \mathcal{B C}(\Pi) \ldots \text { partial solution } \\
\mathcal{C C}(\Pi):=\bigcap \mathcal{A S}(\Pi) \ldots \text { cautious consequences }
\end{gathered}
$$

## Part 1 Weighted Faceted Navigation

## Systematic Faceted Navigation

$\Pi: \quad$ a|b. $\quad c \mid d \leftarrow \mathrm{~b}$. e.

## Systematic Faceted Navigation

$\Pi: \quad$ a $|\mathrm{b} . \quad \mathrm{c}| \mathrm{d} \leftarrow \mathrm{b} . \quad$ e.

Facets: $\mathcal{F}(\Pi)=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \overline{\mathrm{a}}, \overline{\mathrm{b}}, \overline{\mathrm{c}}, \overline{\mathrm{d}}\}$

ANSWER 1: a, e
ANSWER 2: b, c, e
ANSWER 3: b, d, e

## Systematic Faceted Navigation

$\Pi: \quad a|b . \quad c| d \leftarrow b . \quad$ e.

Facets: $\mathcal{F}(\Pi):=\underbrace{\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}}_{\mathcal{F}^{+}(\Pi):=\mathcal{B C}(\Pi) \backslash \mathcal{C C}(\Pi) \quad \mathcal{F}^{-}(\Pi):=\left\{\bar{\alpha} \mid \alpha \in \mathcal{F}^{+}(\Pi)\right\}}$
ANSWER 1: a, e
ANSWER 2: b, c, e
ANSWER 3: b, d, e

## Systematic Faceted Navigation

$\Pi: \quad a|b . \quad c| d \leftarrow b . \quad$ e.
Facets: $\mathcal{F}(\Pi)=\{a, b, c, d, \bar{a}, \bar{b}, \bar{c}, \bar{d}\}$
Routes: $\Delta^{\Pi}:=\left\{\left\langle\mathrm{f}_{0}, \ldots, \mathrm{f}_{\mathrm{n}}\right\rangle \mid \mathrm{f}_{\mathrm{i}} \in \mathcal{F}(\Pi), 0 \leq \mathrm{i} \leq \mathrm{n}\right\} \cup\{\epsilon\}$

ANSWER 1: a, e
ANSWER 2: b, c, e
ANSWER 3: b,d,e

## Systematic Faceted Navigation

$\Pi: \quad$ a $\mid$ b. $\quad c \mid d \leftarrow b . \quad$ e.

Facets: $\mathcal{F}(\Pi)=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \overline{\mathrm{a}}, \overline{\mathrm{b}}, \overline{\mathrm{c}}, \overline{\mathrm{d}}\}$
Routes: $\Delta^{\Pi}:=\left\{\left\langle\mathrm{f}_{0}, \ldots, \mathrm{f}_{\mathrm{n}}\right\rangle \mid \mathrm{f}_{\mathrm{i}} \in \mathcal{F}(\Pi), 0 \leq \mathrm{i} \leq \mathrm{n}\right\} \cup\{\epsilon\}$

ANSWER 1: a, e
ANSWER 2: b, c, e
ANSWER 3: b, d, e


## Systematic Faceted Navigation

$\Pi: \quad$ a $\mid$ b. $\quad c \mid d \leftarrow b . \quad$ e.
ANSWER 1: a, e
Facets: $\mathcal{F}(\Pi)=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \overline{\mathrm{a}}, \overline{\mathrm{b}}, \overline{\mathrm{c}}, \overline{\mathrm{d}}\}$
Routes: $\Delta^{\Pi}:=\left\{\left\langle\mathrm{f}_{0}, \ldots, \mathrm{f}_{\mathrm{n}}\right\rangle \mid \mathrm{f}_{\mathrm{i}} \in \mathcal{F}(\Pi), 0 \leq \mathrm{i} \leq \mathrm{n}\right\} \cup\{\epsilon\}$

ANSWER 2: b, c, e
ANSWER 3: b, d, e


## Systematic Faceted Navigation

$\Pi: \quad a|b . \quad c| d \leftarrow b . \quad$ e.
Facets: $\mathcal{F}(\Pi)=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \overline{\mathrm{a}}, \overline{\mathrm{b}}, \overline{\mathrm{c}}, \overline{\mathrm{d}}\}$
Routes: $\Delta^{\Pi}:=\left\{\left\langle\mathrm{f}_{0}, \ldots, \mathrm{f}_{\mathrm{n}}\right\rangle \mid \mathrm{f}_{\mathrm{i}} \in \mathcal{F}(\Pi), 0 \leq \mathrm{i} \leq \mathrm{n}\right\} \cup\{\epsilon\}$

| ANSWER 1: $\quad \mathrm{a}, \mathrm{e}$ |
| :--- |
| ANSWER 2: |
| b, c, e |
| ANSWER 3: |



## Systematic Faceted Navigation

$\Pi: \quad a|b . \quad c| d \leftarrow b . \quad$ e.

| Facets: $\mathcal{F}(\Pi)=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \overline{\mathrm{a}}, \overline{\mathrm{b}}, \overline{\mathrm{c}}, \overline{\mathrm{d}}\}$ | ANSWER 1: $\mathrm{a}, \mathrm{e}$ |
| :--- | :--- |
| $\underline{\text { Routes: }} \Delta^{\Pi}:=\left\{\left\langle\mathrm{f}_{0}, \ldots, \mathrm{f}_{\mathrm{n}}\right\rangle \mid \mathrm{f}_{\mathrm{i}} \in \mathcal{F}(\Pi), 0 \leq \mathrm{i} \leq \mathrm{n}\right\} \cup\{\epsilon\}$ | ANSWER 2: $\quad \mathrm{b}, \mathrm{c}, \mathrm{e}$ |



## Systematic Faceted Navigation

$\Pi: \quad$ a $\mid$ b. $\quad c \mid d \leftarrow$ b. $\quad$ e.
Facets: $\mathcal{F}(\Pi)=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \overline{\mathrm{a}}, \overline{\mathrm{b}}, \overline{\mathrm{c}}, \overline{\mathrm{d}}\}$
Routes: $\Delta^{\Pi}:=\left\{\left\langle\mathrm{f}_{0}, \ldots, \mathrm{f}_{\mathrm{n}}\right\rangle \mid \mathrm{f}_{\mathrm{i}} \in \mathcal{F}(\Pi), 0 \leq \mathrm{i} \leq \mathrm{n}\right\} \cup\{\epsilon\}$

ANSWER 1: $\quad \mathrm{a}, \mathrm{e}$
ANSWER 2: b, c, e
ANSWER 3: b, d, e


## What is the effect of taking a certain navigation step?

Can we somehow characterize sub-spaces beforehand?

[1] Johannes Klaus Fichte, SAG, Dominik Rusovac. Rushing and Strolling among Answer Sets - Navigation Made Easy Proceedings of the 36th AAAI Conference on Artificial Intelligence (AAAI 2022), 2022.

## What is the effect of taking a certain navigation step?

Can we somehow characterize sub-spaces beforehand?


# 8 Let's do some counting! 

Quantifying effects of navigation steps

## The Weight of a Facet

## Definition (weighting function)

We call $\#:\left\{\Pi^{\delta} \mid \delta \in \Delta^{\Pi}\right\} \rightarrow \mathbb{N}$ weighting function, whenever $\#\left(\Pi^{\delta}\right)>0$, if $|\mathcal{A S}(\Pi)| \geq 2$.

## Definition (weight)

Let $\delta \in \Delta^{\Pi}, \mathrm{f} \in \mathcal{F}(\Pi)$ and $\delta^{\prime}$ be a redirection of $\delta$ w.r.t. f. The weight of f w.r.t. $\#, \Pi^{\delta}$ and $\delta^{\prime}$ is defined as:

$$
\omega_{\#}\left(\mathrm{f}, \Pi^{\delta}, \delta^{\prime}\right):= \begin{cases}\#\left(\Pi^{\delta}\right)-\#\left(\Pi^{\delta^{\prime}}\right), & \text { if }\langle\delta, \mathrm{f}\rangle \notin \Delta_{\mathrm{s}}^{\Pi} \text { and } \delta^{\prime} \neq \epsilon ; \\ \#\left(\Pi^{\delta}\right)-\#\left(\Pi^{\delta \delta, \mathrm{f}\rangle}\right), & \text { otherwise. }\end{cases}
$$

## The Weight of a Facet

## Definition (weight)

Let $\delta \in \Delta^{\Pi}, \mathrm{f} \in \mathcal{F}(\Pi)$ and $\delta^{\prime}$ be a redirection of $\delta$ w.r.t. f. The weight of f w.r.t. $\#, \Pi^{\delta}$ and $\delta^{\prime}$ is defined as:

$$
\omega_{\#}\left(\mathrm{f}, \Pi^{\delta}, \delta^{\prime}\right):= \begin{cases}\#\left(\Pi^{\delta}\right)-\#\left(\Pi^{\delta^{\prime}}\right), & \text { if }\langle\delta, \mathrm{f}\rangle \notin \Delta_{\mathrm{s}}^{\Pi} \text { and } \delta^{\prime} \neq \epsilon ; \\ \#\left(\Pi^{\delta}\right)-\#\left(\Pi^{\delta \delta, f\rangle}\right), & \text { otherwise. }\end{cases}
$$

Effects:


## Which Weighting Function? Absolute vs. Relative Weights

Natural choice?

## Which Weighting Function? Absolute vs. Relative Weights

## Natural choice?

- Absolute Weight:

Count Answer Sets with $\omega_{\#_{\mathcal{A S}}}$

$$
\begin{aligned}
& \mathcal{A S}(\Pi)=\{\{\mathrm{a}, \mathrm{e}\},\{\mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{b}, \mathrm{~d}, \mathrm{e}\}\} \\
& \omega_{\# \mathcal{A S}}(\mathrm{~b}, \Pi, \epsilon)=1 \\
& \mathcal{A S}\left(\Pi^{\langle\mathrm{b}\rangle}\right)=\{\{\mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{b}, \mathrm{~d}, \mathrm{e}\}\}
\end{aligned}
$$

## Which Weighting Function? Absolute vs. Relative Weights

## Natural choice?

- Absolute Weight:

Count Answer Sets with $\omega_{\#_{\mathcal{A S}}}$
Counting answer sets is hard $(3$ [3]

$$
\begin{gathered}
\mathcal{A S}(\Pi)=\{\{\mathrm{a}, \mathrm{e}\},\{\mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{b}, \mathrm{~d}, \mathrm{e}\}\} \\
\omega_{\# \mathcal{A S}}(\mathrm{~b}, \Pi, \epsilon)=1 \\
\mathcal{A S}\left(\Pi^{\langle\mathrm{b}\rangle}\right)=\{\{\mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{b}, \mathrm{~d}, \mathrm{e}\}\}
\end{gathered}
$$

[3] Johannes K Fichte, Markus Hecher, Michael Morak, and Stefan Woltran. Answer set solving with bounded treewidth revisited. In LPNMR 2017.

## Which Weighting Function? Absolute vs. Relative Weights

## Natural choice?

- Absolute Weight:

Count Answer Sets with $\omega_{\#_{\mathcal{A S}}}$
Counting answer sets is hard © [3]
Relative Weights:
cheaper methods to quantify effects

$$
\begin{aligned}
& \mathcal{A} \mathcal{S}(\Pi)=\{\{\mathrm{a}, \mathrm{e}\},\{\mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{b}, \mathrm{~d}, \mathrm{e}\}\} \\
& \omega_{\# \mathcal{S}}(\mathrm{~b}, \Pi, \epsilon)=1 \\
& \mathcal{A S}\left(\Pi^{(\mathrm{b}\rangle}\right)=\{\{\mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{b}, \mathrm{~d}, \mathrm{e}\}\}
\end{aligned}
$$

- Count Supported Models with $\omega_{\# \mathcal{S}}$
[3] Johannes K Fichte, Markus Hecher, Michael Morak, and Stefan Woltran. Answer set solving with bounded treewidth revisited. In LPNMR 2017.


## Which Weighting Function? Absolute vs. Relative Weights

## Natural choice?

- Absolute Weight:

Count Answer Sets with $\omega_{\#_{\mathcal{A S}}}$

## Counting answer sets is hard $\oplus$ [fichte2017answer]

## Relative Weights:

cheaper methods to quantify effects

- Count Supported Models with $\omega_{\# s}$
- Count Facets with $\omega_{\# \mathcal{F}}$

$$
\mathcal{A S}(\Pi)=\{\{\mathrm{a}, \mathrm{e}\},\{\mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{b}, \mathrm{~d}, \mathrm{e}\}\}
$$

$$
\omega_{\# \mathcal{F}}(\mathrm{~b}, \Pi, \epsilon)=4 \downarrow
$$

$$
\mathcal{A S}\left(\Pi^{\langle\mathrm{b}\rangle}\right)=\{\{\mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{b}, \mathrm{~d}, \mathrm{e}\}\}
$$

$$
\begin{gathered}
\mathcal{A S}(\Pi)=\{\{\mathrm{a}, \mathrm{e}\},\{\mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{b}, \mathrm{~d}, \mathrm{e}\}\} \\
\omega_{\# \mathcal{F}}(\overline{\mathrm{c}}, \Pi, \epsilon)=2 \\
\mathcal{A S}\left(\Pi^{(\bar{c})}\right)=\{\{\mathrm{a}, \mathrm{e}\},\{\mathrm{b}, \mathrm{~d}, \mathrm{e}\}\}
\end{gathered}
$$

[3] Johannes K Fichte, Markus Hecher, Michael Morak, and Stefan Woltran. Answer set solving with bounded treewidth revisited. In LPNMR

## Quantitative Arguments

|  | 2 |  | 5 |  | 1 |  | 9 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 |  |  | 2 |  | 3 |  |  | 6 |
|  | 3 |  |  | 6 |  |  | 7 |  |
| 6 | 6 | 1 |  | 2 | $2 \mid 6$ |  |  |  |
| 5 | 4 |  |  | 2 |  | 8 | 1 | 9 |
|  |  |  |  | $2 \mid 5$ | $2 \mid 5$ | 7 |  |  |
|  | 9 |  |  | 3 |  |  | 8 |  |
| 2 |  |  | 8 |  | 4 |  |  | 7 |
|  | 1 |  | 9 |  | 7 |  | 6 |  |

How to solve this Sudoku as quick as possible?


Which moves (queens) have the least $(1 / 4) /$ most $(3 / 4)$ impact?

## Part 2 Incremental Answer Set Counting

## Counting Efficiently via Knowledge Compilation [4]

$\Pi_{1}=\{\mathrm{a} \leftarrow \mathrm{b} ., \mathrm{b} \leftarrow, \mathrm{c} \leftarrow \mathrm{c}\}$ transform to sd-DNNF $\Phi_{\Pi_{1}}=\left(\left(\mathrm{x}_{3} \wedge \neg \mathrm{c}\right) \vee\left(\neg \mathrm{x}_{3} \wedge \mathrm{c}\right)\right) \wedge\left(\neg \mathrm{x}_{1} \wedge \neg \mathrm{x}_{2} \wedge \neg \mathrm{x}_{5} \wedge \mathrm{a} \wedge \mathrm{b}\right)$


Figure 2: Counting graph $\mathcal{G}_{\Pi_{1}}^{\langle\sim \mathrm{c}\rangle}$ labeled with evaluations.
[4] Johannes Klaus Fichte, SAG, Markus Hecher, Dominik Rusovac. IASCAR: Incremental Answer Set Counting by Anytime Refinement; LPNMR 2022, honorable mention.

## Answer Sets versus Supported Models

$\underline{\text { Logic program } \Pi \text { : }}$

$$
\begin{array}{ll}
\mathrm{a} \leftarrow \mathrm{~b} & \mathrm{~b} \leftarrow \mathrm{a} \\
\mathrm{c} \leftarrow \neg \mathrm{~d} & \mathrm{~d} \leftarrow \neg \mathrm{c} \\
\mathrm{a} \leftarrow \mathrm{c}, \mathrm{e} & \mathrm{e} \leftarrow \mathrm{c}
\end{array}
$$

$$
\begin{array}{lrl}
\text { Answer sets : } & \mathcal{A} \mathcal{S}(\Pi)=\{\{\mathrm{a}, \mathrm{~b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{d}\}\} \\
\text { Supported models: } & \mathcal{S}(\Pi)=\{\{\mathrm{a}, \mathrm{~b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{d}\},\{\mathrm{a}, \mathrm{~b}, \mathrm{~d}\}\}
\end{array}
$$

## Answer Sets versus Supported Models

## Completion:

| $\mathrm{a} \leftrightarrow \mathrm{b} \vee(\mathrm{c} \wedge \mathrm{e})$ | $\mathrm{b} \leftrightarrow \mathrm{a}$ |
| :--- | :--- |
| $\mathrm{c} \leftrightarrow \neg \mathrm{d}$ | $\mathrm{d} \leftrightarrow \neg \mathrm{c}$ |
| $\mathrm{e} \leftrightarrow \mathrm{c}$ |  |

Answer sets :
Supported models:

$$
\mathcal{A S}(\Pi)=\{\{\mathrm{a}, \mathrm{~b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{d}\}\}
$$

models of completion

## Answer Sets versus Supported Models

## Logic program $\Pi$ :

$$
\begin{array}{ll}
\mathrm{a} \leftarrow \mathrm{~b} & \mathrm{~b} \leftarrow \mathrm{a} \\
\mathrm{c} \leftarrow \neg \mathrm{~d} & \mathrm{~d} \leftarrow \neg \mathrm{c} \\
\mathrm{a} \leftarrow \mathrm{c}, \mathrm{e} & \mathrm{e} \leftarrow \mathrm{c}
\end{array}
$$

## Answer Sets versus Supported Models

## Logic program $\Pi$ :

$$
\begin{array}{ll}
\mathrm{a} \leftarrow \mathrm{~b} & \mathrm{~b} \leftarrow \mathrm{a} \\
\mathrm{c} \leftarrow \neg \mathrm{~d} & \mathrm{~d} \leftarrow \neg \mathrm{c} \\
\mathrm{a} \leftarrow \mathrm{c}, \mathrm{e} & \mathrm{e} \leftarrow \mathrm{c}
\end{array}
$$

Answer sets : $\quad \mathcal{A} \mathcal{S}(\Pi)=\{\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{d}\}\}$
Supported models : $\quad \mathcal{S}(\Pi)=\{\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{d}\},\{\mathrm{a}, \mathrm{b}, \mathrm{d}\}\}$
In general $\mathcal{A} \mathcal{S}(\Pi) \subseteq \mathcal{S}(\Pi)$

## Positive dependency graph:



Mismatch caused by cycle $\{a, b\}$ with no external support $\{c, e\}$

## Answer Sets versus Supported Models

## Logic program $\Pi$ :

| $\mathrm{a} \leftarrow \mathrm{b}$ | $\mathrm{b} \leftarrow \mathrm{a}$ |
| :--- | :--- |
| $\mathrm{c} \leftarrow \neg \mathrm{d}$ | $\mathrm{d} \leftarrow \neg \mathrm{c}$ |
| $\mathrm{a} \leftarrow \mathrm{c}, \mathrm{e}$ | $\mathrm{e} \leftarrow \mathrm{c}$ |

Answer sets : $\quad \mathcal{A} \mathcal{S}(\Pi)=\{\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{d}\}\}$
Supported models : $\quad \mathcal{S}(\Pi)=\{\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{d}\},\{\mathrm{a}, \mathrm{b}, \mathrm{d}\}\}$
In general $\mathcal{A} \mathcal{S}(\Pi) \subseteq \mathcal{S}(\Pi)$

## Positive dependency graph:



## Identifying Cycles without External Support

 b
d
$\mathcal{A S}(\Pi)=\{\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{d}\}\}$
$\mathcal{S}(\Pi)=\{\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{e}\},\{\mathrm{d}\}, \underline{\{\mathrm{a}, \mathrm{b}, \mathrm{d}\}}\}$

## Unsupported constraint

- Rule of form $\leftarrow \mathrm{c}_{0}, \ldots, \mathrm{c}_{\mathrm{n}}, \neg \mathrm{S}_{0}, \ldots, \neg \mathrm{~S}_{\mathrm{m}}$
cycle
external support
- Example: $\mathrm{r}=\leftarrow \mathrm{a}, \mathrm{b}, \neg \mathrm{c}, \neg \mathrm{e}$
- Pruning mismatch: $\mathcal{S}(\Pi \cup\{\mathrm{r}\})=\mathcal{A S}(\Pi)$
- Identifying mismatch: $\mathcal{S}\left(\Pi^{\mathrm{B}(\mathrm{r})}\right)=\mathcal{S}(\Pi) \backslash \mathcal{A} \mathcal{S}(\Pi)$ where $\mathrm{B}(\mathrm{r})=\{\mathrm{a}, \mathrm{b}, \neg \mathrm{c}, \neg \mathrm{e}\}$


## Incremental Answer Set Counting with Anytime Refinement



## Incremental Answer Set Counting with Anytime Refinement



Counting by $\mathrm{a}_{\mathrm{d}}^{\mathrm{L}}:=\sum_{\mathrm{i}=0}^{\mathrm{d}}(-1)^{\mathrm{i}} \sum_{\Gamma \in \Lambda_{\mathrm{i}}(\Pi)}\left|\mathcal{S}\left(\Pi^{\mathrm{LuB}(\Gamma)}\right)\right|$ with alternation depth d

## Incremental Answer Set Counting with Anytime Refinement



Counting by $\mathrm{a}_{\mathrm{d}}^{\mathrm{L}}:=\sum_{\mathrm{i}=0}^{\mathrm{d}}(-1)^{\mathrm{i}} \sum_{\Gamma \in \Lambda_{\mathrm{i}}(\Pi)}\left|\mathcal{S}\left(\Pi^{\mathrm{L} \cup B(\Gamma)}\right)\right|$ with alternation depth d

1. Include all supported models under assumptions L by $\left|\mathcal{S}\left(\Pi^{\mathrm{L}}\right)\right|$

## Incremental Answer Set Counting with Anytime Refinement



Counting by $\mathrm{a}_{\mathrm{d}}^{\mathrm{L}}:=\sum_{\mathrm{i}=0}^{\mathrm{d}}(-1)^{\mathrm{i}} \sum_{\Gamma \in \Lambda_{\mathrm{i}}(\Pi)}\left|\mathcal{S}\left(\Pi^{\mathrm{LuB}(\Gamma)}\right)\right|$ with alternation depth d

1. Include all supported models under assumptions L by $\left|\mathcal{S}\left(\Pi^{\mathrm{L}}\right)\right|$
2. Exclude mismatches identified via singleton cycles by $-\sum_{\Gamma \in \Lambda_{1}(\Pi)}\left|\mathcal{S}\left(\Pi^{\mathrm{LuB}(\Gamma)}\right)\right|$

## Incremental Answer Set Counting with Anytime Refinement



$$
\mathcal{S}(\Pi)=\{\{\mathrm{d}\},\{\mathrm{d}, \mathrm{e}, \mathrm{f}\},\{\mathrm{a}, \mathrm{~b}, \mathrm{~d}\},\{\mathrm{a}, \mathrm{~b}, \mathrm{c}\},\{\mathrm{a}, \mathrm{~b}, \mathrm{c}, \mathrm{e}, \mathrm{f}\},\{\mathrm{a}, \mathrm{~b}, \mathrm{~d}, \mathrm{e}, \mathrm{f}\}\}
$$

Counting by $\mathrm{a}_{\mathrm{d}}^{\mathrm{L}}:=\sum_{\mathrm{i}=0}^{\mathrm{d}}(-1)^{\mathrm{i}} \sum_{\Gamma \in \Lambda_{\mathrm{i}}(\Pi)}\left|\mathcal{S}\left(\Pi^{\mathrm{LuB}(\Gamma)}\right)\right|$ with alternation depth d

1. Include all supported models under assumptions L by $\left|\mathcal{S}\left(\Pi^{\mathrm{L}}\right)\right|$
2. Exclude mismatches identified via singleton cycles by $-\sum_{\Gamma \in \Lambda_{1}(\Pi)}\left|\mathcal{S}\left(\Pi^{\mathrm{LuB}(\Gamma)}\right)\right|$
3. Include mistakenly excluded models by $+\sum_{\Gamma \in \Lambda_{2}(\Pi)}\left|\mathcal{S}\left(\Pi^{\mathrm{LuB}(\Gamma)}\right)\right|$ via combining 2 cycles $\vdots$

## Incremental Answer Set Counting with Anytime Refinement



$$
\mathcal{S}(\Pi)=\{\{\mathrm{d}\},\{\mathrm{d}, \mathrm{e}, \mathrm{f}\},\{\mathrm{a}, \mathrm{~b}, \mathrm{~d}\},\{\mathrm{a}, \mathrm{~b}, \mathrm{c}\},\{\mathrm{a}, \mathrm{~b}, \mathrm{c}, \mathrm{e}, \mathrm{f}\},\{\mathrm{a}, \mathrm{~b}, \mathrm{~d}, \mathrm{e}, \mathrm{f}\}\}
$$

## Example

$$
\begin{aligned}
& \mathrm{a}_{1}^{\{\mathrm{d}\}}=\left|\mathcal{S}\left(\Pi^{\{\mathrm{d}\}}\right)\right|-\left|\mathcal{S}\left(\Pi^{\{\mathrm{d}, \mathrm{a}, \mathrm{~b}, \neg \mathrm{c}, \neg \mathrm{~g}\}}\right)\right|-\left|\mathcal{S}\left(\Pi^{\{\mathrm{d}, \mathrm{e}, \mathrm{f}, \neg \mathrm{~g}\}}\right)\right|=0 \neq\left|\mathcal{A S}\left(\Pi^{\{\mathrm{d}\}}\right)\right|=1 \\
& \mathrm{a}_{2}^{\{\mathrm{d}\}}=\mathrm{a}_{1}^{\{\mathrm{d}\}}+\left|\mathcal{S}\left(\Pi^{\{\mathrm{d}, \mathrm{a}, \mathrm{~b}, \neg \mathrm{c}, \neg \mathrm{~g}, \mathrm{e}, \mathrm{f}\}}\right)\right|=\left|\mathcal{A} \mathcal{S}\left(\Pi^{\{\mathrm{d}\}}\right)\right|=1
\end{aligned}
$$

## Incremental Answer Set Counting with Anytime Refinement



## Example

$$
\begin{aligned}
& \mathrm{a}_{1}^{\{\mathrm{d}\}}=\left|\mathcal{S}\left(\Pi^{\{\mathrm{d}\}}\right)\right|-\left|\mathcal{S}\left(\Pi^{\{\mathrm{d}, \mathrm{a}, \mathrm{~b}, \mathrm{c},-\mathrm{g}\}}\right)\right|-\left|\mathcal{S}\left(\Pi^{\{\mathrm{d}, \mathrm{e}, \mathrm{f}, \mathrm{qg}\}}\right)\right|=0 \neq\left|\mathcal{A} \mathcal{S}\left(\Pi^{\{\mathrm{d}\}}\right)\right|=1 \\
& \mathrm{a}_{2}^{\{\mathrm{d}\}}=\mathrm{a}_{1}^{\{\mathrm{d}\}}+\left|\mathcal{S}\left(\Pi^{\{\mathrm{d}, \mathrm{a}, \mathrm{~b}, \neg \mathrm{c}, \mathrm{q}, \mathrm{e}, \mathrm{f}\}}\right)\right|=\left|\mathcal{A} \mathcal{S}\left(\Pi^{\{\mathrm{d}\}}\right)\right|=1
\end{aligned}
$$

Theorem (Exact Counting)
If $\mathrm{d}=|\operatorname{cycles}(\Pi)|$, then $\mathrm{a}_{\mathrm{d}}^{\mathrm{L}}=\left|\mathcal{A S}\left(\Pi^{\mathrm{L}}\right)\right|$.

$$
\begin{aligned}
& \text { Theorem (Early Termination) } \\
& \text { If } \mathrm{a}_{\mathrm{i}}^{\mathrm{L}}=\mathrm{a}_{\mathrm{i}+1}^{\mathrm{L}} \text {, then } \mathrm{a}_{\mathrm{i}}^{\mathrm{L}}=\left|\mathcal{A} \mathcal{S}\left(\Pi^{\mathrm{L}}\right)\right|
\end{aligned}
$$

## Architecture IASCAR Tool



## Part 3 Diverse Answer Sets

Reworking Methods [5]

[5] Elisa Böhl, SAG. Tunas - Fishing for Diverse Answer Sets: A Multi-Shot Trade up Strategy; LPNMR 2022.

Reworking Methods [5]

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## Reworking Methods [5]

## Problem Definition


[5] Elisa Böhl, SAG. Tunas - Fishing for Diverse Answer Sets: A Multi-Shot Trade up Strategy; LPNMR 2022.

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Reworking Methods [5]

## Problem Definition

$m$ deletion
$-\left|S \backslash S^{\prime}\right| \leq m$

## candidates $D$

collection S
[5] Elisa Böhl, SAG. Tunas - Fishing for Diverse Answer Sets: A Multi-Shot Trade up Strategy; LPNMR 2022.

Reworking Methods [5]

## Problem Definition

- | $\mathrm{S}^{\prime}|>|\mathrm{S}|$
$-\left|S \backslash S^{\prime}\right| \leq m$


## $m$ deletion

candidates $D$

$$
m+1 \text { new }
$$ solutions $N$

collection S

[^0]Reworking Methods [5]

## Problem Definition

- |S'|>|S|
- $\left|S \backslash S^{\prime}\right| \leq m$
- $\Delta\left(\mathrm{S}^{\prime}\right) \geq \mathrm{k}$
collection S'

[^1]Reworking Methods [5]

## Problem Definition

- $\left|S^{\prime}\right|>|S|$
- $\left|S \backslash S^{\prime}\right| \leq m$
- $\Delta\left(\mathrm{S}^{\prime}\right) \geq \mathrm{k}$
- NP-complete

[5] Elisa Böhl, SAG. Tunas - Fishing for Diverse Answer Sets: A Multi-Shot Trade up Strategy; LPNMR 2022.


## Tunas - Trade Up Navigation for Answer Sets



## Tunas - Trade Up Navigation for Answer Sets

## Properties

- comparatively fast
- anytime approach
- multi-shot
- additive grounding
- concealment and deletion of atoms
- counting over changing domain


## Part 4 Visual Approach for Solution Space Exploaration

## NEXAS: A Visual Tool for Navigating and Exploring Argumentation Solution Spaces [6]


[6] Raimund Dachselt, SAG, Markus Krötzsch, Julián Méndez, Dominik Rusovac, Mei Yang. NEXAS: A Visual Tool for Navigating and Exploring Argumentation Solution Spaces; COMMA 2022. https://imld.de/nexas

## Summary \& Future Work

Summary:

- Weighted faceted navigation allows to quantitatively explore the solution space
- Incremental answer set counting via knowledge compilation and anytime refinement
- Iterative reworking strategies to comupte diverse answer sets
- Visual exploration of solution space

Future Work:

- Further interesting properties of facet-counting weight
- Measure the quality of diverse solutions
- Generalize visualization approach for answer sets


[^0]:    [5] Elisa Böhl, SAG. Tunas - Fishing for Diverse Answer Sets: A Multi-Shot Trade up Strategy; LPNMR 2022.

[^1]:    [5] Elisa Böhl, SAG. Tunas - Fishing for Diverse Answer Sets: A Multi-Shot Trade up Strategy; LPNMR 2022.

