Exercise 6.1. Let $A_{LBA}$ be the word problem of deterministic linear bounded automata. Show that $A_{LBA}$ is $PSPACE$-complete.

Exercise 6.2. Show that if every $NP$-hard language is also $PSPACE$-hard, then $NP = PSPACE$.

Exercise 6.3. Consider the Japanese game *go-moku* that is played by two players $X$ and $O$ on a $19 \times 19$ board. Players alternately place markers on the board, and the first one to have five of its markers in a row, column, or diagonal wins.

Consider the generalized version of go-moku on an $n \times n$ board. Say that a position of go-moku is a placement of markers on such a board as it could occur during the game, together with a marker which player moves next. Define

$$GM = \{ \langle B \rangle \mid B \text{ is a position of go-moku where } X \text{ has a winning strategy} \}.$$ 

Show that $GM$ is in $PSPACE$.

Exercise 6.4. Show that the universality problem of nondeterministic finite automata

$$\text{ALL}_{\text{NFA}} = \{ \langle A \rangle \mid A \text{ an NFA accepting every valid input} \}$$

is in $PSPACE$.

*Hint:* Prove that, if $L(A) \neq \Sigma^*$ and $A$ has $n$ states, then there exists a word $w \in \Sigma^*$ of length at most $2n$ such that $w \notin L(A)$. Then, use this fact to give a non-deterministic algorithm whose space consumption is polynomially bounded. Finally, apply Savitch's Theorem.

Exercise 6.5. Let $EQ_{REX} = \{ (R, S) \mid R \text{ and } S \text{ are equivalent regular expressions} \}$.

Show that $EQ_{REX}$ is in $PSPACE$.

*Hint:* Adapt the hint of Exercise 6.4 accordingly.