TECHNISCHE UNIVERSITÄT DRESDEN

Artificial Intelligence, Computational Logic

ABSTRACT ARGUMENTATION

Complexity and Equivalences of Argumentation Frameworks

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ICCL Summer School 2016

DRESDEN concept Exzellenz aus Wissenschaft und Kultur

Outline

Decision Problems on AFs

Credulous Acceptance

Cred$_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is a contained in at least one σ-extension of F?

Skeptical Acceptance

Skept$_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is a contained in every σ-extension of F?

If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted[1].

[1] This is only relevant for stable semantics.

Decision Problems on AFs

Credulous Acceptance

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Skeptical Acceptance

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If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted[1].

Hence we are also interested in the following problem:

Skeptically and Credulously accepted

Skept$'_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is a contained in every and at least one σ-extension of F?

[1] This is only relevant for stable semantics.

Further Decision Problems

Verifying an extension

Ver_σ: Given AF $F = (A, R)$ and $S \subseteq A$; is S a σ-extension of F?

Further Decision Problems

Verifying an extension

Ver$_\sigma$: Given AF $F = (A, R)$ and $S \subseteq A$; is S a σ-extension of F?

Does there exist an extension?

Exists$_\sigma$: Given AF $F = (A, R)$; Does there exist a σ-extension for F?

Further Decision Problems

Verifying an extension

Ver_σ: Given AF $F = (A, R)$ and $S \subseteq A$; is S a σ-extension of F?

Does there exist an extension?

Exists_σ: Given AF $F = (A, R)$; Does there exist a σ-extension for F?

Does there exist a nonempty extensions?

$\text{Exists}_\sigma^{\neg\emptyset}$: Does there exist a non-empty σ-extension for F?

Complexity Results (Summary)

Complexity for decision problems in AFs.

σ	Cred_σ	Skept_σ
ground	P-c	P-c
naive	in L	in L
stable	NP-c	co-NP-c
adm	NP-c	trivial
comp	NP-c	P-c
pref	NP-c	Π_2^p-c

σ	Cred_σ	Skept_σ
semi	Σ_2^p-c	Π_2^p-c
stage	Σ_2^p-c	Π_2^p-c
ideal	in Θ_2^p	in Θ_2^p
eager	Π_2^p-c	Π_2^p-c
*ground**	NP-c	co-NP-c
cf2	NP-c	co-NP-c

see [Baroni et al.2011, Coste-Marquis et al.2005, Dimopoulos and Torres1996, Dung1995, Dunne2008, Dunne and Bench-Capon2002, Dunne and Bench-Capon2004, Dunne and Caminada2008, Dvořák et al.2011, Dvořák and Woltran2010a, Dvořák and Woltran2010b]

Intractable problems in Abstract Argumentation

Most problems in Abstract Argumentation are computationally intractable, i.e. at least NP-hard. To show intractability for a specific reasoning problem we follow the schema given below:

Goal: Show that a reasoning problem is NP-hard.

Method: Reducing the NP-hard SAT problem to the reasoning problem.

- Consider an arbitrary CNF formula Φ
- Give a reduction that maps Φ to an Argumentation Framework F_Φ containing an argument Φ.
- Show that Φ is satisfiable iff the argument Φ is accepted.

Canonical Reduction

Definition

For $\Phi = \bigwedge_{i=1}^{m} l_{i1} \vee l_{i2} \vee l_{i3}$ over atoms Z, build $F_\Phi = (A_\Phi, R_\Phi)$ with

$$\begin{aligned} A_\Phi &= Z \cup \bar{Z} \cup \{C_1, \ldots, C_m\} \cup \{\Phi\} \\ R_\Phi &= \{(z, \bar{z}), (\bar{z}, z) \mid z \in Z\} \cup \{(C_i, \Phi) \mid i \in \{1, \ldots, m\}\} \cup \\ &\quad \{(z, C_i) \mid i \in \{1, \ldots, m\}, z \in \{l_{i1}, l_{i2}, l_{i3}\}\} \cup \\ &\quad \{(\bar{z}, C_i) \mid i \in \{1, \ldots, m\}, \neg z \in \{l_{i1}, l_{i2}, l_{i3}\}\} \end{aligned}$$

Canonical Reduction

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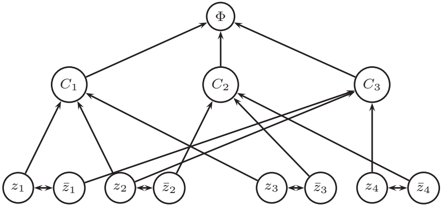
Example

Let $\Phi = (z_1 \vee z_2 \vee z_3) \wedge (\neg z_2 \vee \neg z_3 \vee \neg z_4) \wedge (\neg z_1 \vee z_2 \vee z_4)$.



Canonical Reduction: CNF $\Rightarrow$ AF (ctd.)

Theorem

The following statements are equivalent:

1. Φ is satisfiable
2. F_Φ has an admissible set containing Φ
3. F_Φ has a complete extension containing Φ
4. F_Φ has a preferred extension containing Φ
5. F_Φ has a stable extension containing Φ

Complexity Results

Theorem

1. Cred_{stable} is NP-complete
2. Cred_{adm} is NP-complete
3. Cred_{comp} is NP-complete
4. Cred_{pref} is NP-complete

Proof.

(1) The hardness is immediate by the last theorem.
For the NP-membership we use the following guess & check algorithm:

- Guess a set $E \subseteq A$
- verify that E is stable
    - for each $a, b \in E$ check $(a, b) \notin R$
    - for each $a \in A \setminus E$ check if there exists $b \in E$ with $(b, a) \in R$

As this algorithm is in polynomial time we obtain NP-membership. □

Outline

Argumentation Systems

Tools with web-interface

- ASPARTIX `http://rull.dbai.tuwien.ac.at:8080/ASPARTIX/`
- ConArg `http://www.dmi.unipg.it/conarg/`

Further Systems

- See first International Competition on Computational Models of Argumentation (ICCMA) `http://argumentationcompetition.org` and [Charwat et al., 2015].

ICCMA'17

The Second International Competition on Computational Models of Argumentation (ICCMA'17) `http://www.dbai.tuwien.ac.at/iccma17/`

Outline

Intertranslatability of Semantics

Motivation

- Advanced engine for semantics σ' available but we want to evaluate F wrt. semantics σ
- Transform F into F' s.t. evaluating F' wrt. σ' allows for an easy reconstruction of σ-extensions of F
- If Transformation is efficiently computable, this is a more successful approach than implementing a distinguished algorithm for σ

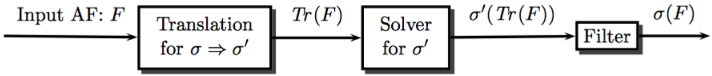
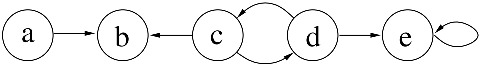


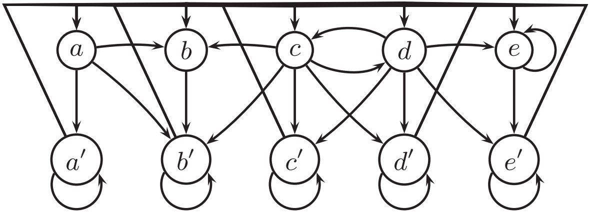
Figure: Solver for a semantic σ, using a translation for $\sigma \Rightarrow \sigma'$

Translating Semantics [Dvořák and Woltran, 2011]

Translation τ for embedding stable into admissible / complete semantics.



$\Downarrow$

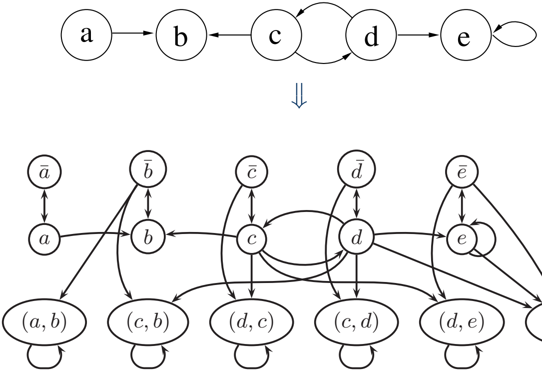


Result:

For each AF F, $stable(F) = \sigma(\tau(F)) \setminus \{\emptyset\}$ with $\sigma \in \{adm, comp\}$.

Translating Semantics

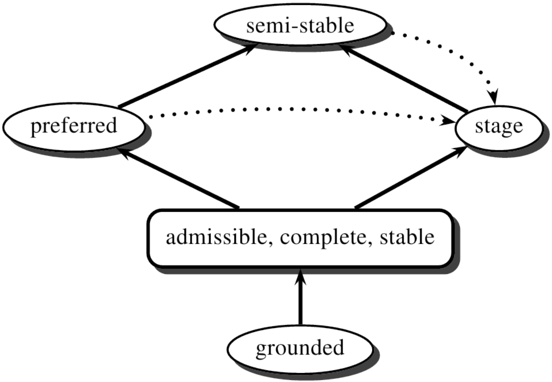
Translating admissible to stable / semi-stable/ stage semantics.



Result:

Tr_α is a faithful translation for $adm \Rightarrow stable$.

Translating Semantics (big picture)



Intertranslatability w.r.t. (weakly) faithful translations

Outline

Motivation

- Argumentation is a dynamic reasoning process.
- During the process the participants come up with new arguments.
  - Which effects causes additional information wrt. a semantics?
  - Which information does not contribute to the results?

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- Two AFs F and G are strongly equivalent (wrt. a semantics σ) iff $F \cup H$ and $G \cup H$ have the same σ-extensions for each AF H.
  - One can savely replace an AF by a strongly equivalent one without changing its extensions.

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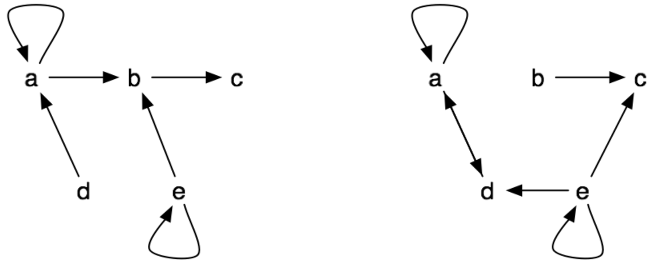
    - One can savely replace an AF by a strongly equivalent one without changing its extensions.
- In a negotiation between two agents: SE allows to characterize situations where the two agents have an equivalent view of the world which is moreover robust to additional information.

Outline

Standard Equivalence of AFs

Example



- AFs F and G are equivalent (wrt. stable semantics).
- $stable(F) = stable(G) = \emptyset$

Standard Equivalence

Intuitively, semantically equivalent AF yield the same result when applying the semantic operators.

Standard Equivalence [Oikarinen and Woltran, 2011]

AFs F and G are standard equivalent wrt. a given semantic σ ($F \equiv^{\sigma} G$), iff they posses the same extensions under the semantic σ.

Results wrt. Standard Equivalence

For any AFs F and G we have

- $adm(F) = adm(G) \implies pref(F) = pref(G)$
- $adm(F) = adm(G) \implies ideal(F) = ideal(G)$
- $comp(F) = comp(G) \implies pref(F) = pref(G)$
- $comp(F) = comp(G) \implies ground(F) = ground(G)$
- $comp(F) = comp(G) \implies ideal(F) = ideal(G)$
- $adm(F) = adm(G)$ and $semi(F) = semi(G) \implies eager(F) = eager(G)$

Standard Equivalence of AFs

Intuitively, semantically equivalent AF yield the same result when applying the semantic operators.

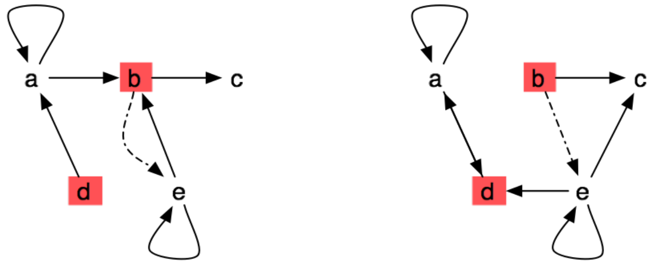
Standard Equivalence

AFs F and G are standard equivalent wrt. a given semantic σ ($F \equiv^\sigma G$), iff they posses the same extensions under the semantic σ.

- Appropriate from a static view point.
- But AA is not static, rather a highly dynamic process, they are expanded over time (e.g., during an analysis phase)

Strong Equivalence of AFs

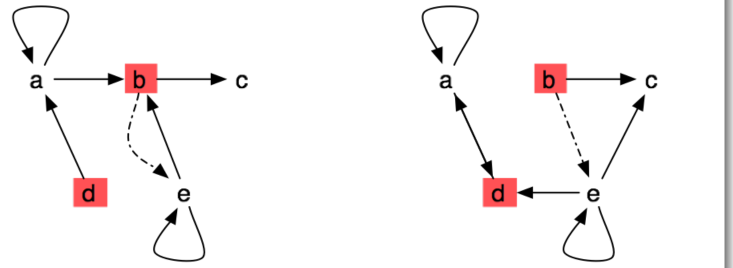
Example



- $stable(F \cup H) = stable(G \cup H) = \{\{b, d\}\}$.

Strong Equivalence of AFs

Example



- Goal: identify redundant attacks:
  - Find attacks which do not contribute in the evaluation of F, no matter how F is extended

$\Longrightarrow$ Define kernel of an AF (remove redundant attacks)

$\Longrightarrow$ Checking for strong equivalence reduces to check syntactic equivalence

Motivation ctd.

- Identification of redundant attacks is important in choosing an appropriate semantics.
- Caminada and Amgoud outlined in [Caminada and Amgoud, 2007] that the interplay between how a framework is built and which semantics is used to evaluate the framework is crucial in order to obtain useful results when the (claims of the) arguments selected by the chosen semantics are collected together.
- Knowledge about redundant attacks (wrt. a particular semantics) might help to identify unsuitable such combinations.
- Strong equivalence has been analyzed for many semantics in [Oikarinen and Woltran, 2010].
- Naive-based semantics naive, stage and *cf2* have been analyzed in [Gaggl and Woltran, 2013].

Outline

Strong Equivalence (SE)

Respecting dynamic aspects, one needs to develop stronger equivalent notions.

Strong Equivalence [Oikarinen and Woltran, 2010]

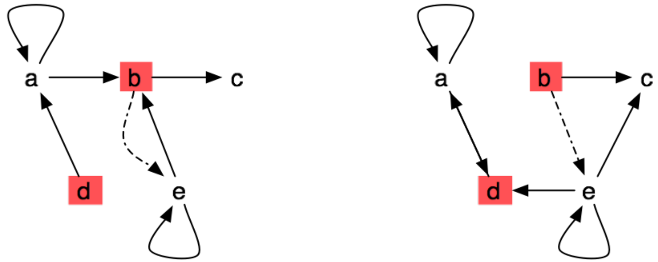
Two AFs F and G are strongly equivalent to each other wrt. a semantics σ, in symbols $F \equiv_s^\sigma G$, iff for each AF H, $\sigma(F \cup H) = \sigma(G \cup H)$.

- By definition $F \equiv_s^\sigma G$ implies $\sigma(F) = \sigma(G)$
- The AF H represents possible (dynamic) growth of F and G

Outline

SE wrt. Stable Semantics

Example



s-kernel

For an AF $F = (A, R)$ we define the s-kernel of F as $F^{sk} = (A, R^{sk})$ where

$$R^{sk} = R \setminus \{(a, b) \mid a \neq b, (a, a) \in R\}.$$

SE wrt. Stable Semantics

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SE wrt. Stable Semantics

- For any AF F, $stable(F) = stable(F^{sk})$
- Let F and G be AFs, s.t. $F^{sk} = G^{sk}$. Then, $(F \cup H)^{sk} = (G \cup H)^{sk}$ for each AF H
- For any AFs F and G: $F^{sk} = G^{sk}$ iff $F \equiv_s^{stable} G$

SE wrt. Stable Semantics

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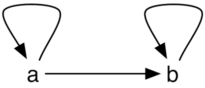
- For any AFs F and G: $F^{sk} = G^{sk}$ iff $F \equiv_s^{stable} G$

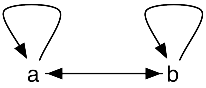
SE wrt. Stage Semantics

- For any AFs F and G: $F^{sk} = G^{sk}$ iff $F \equiv_s^{stage} G$

SE wrt. Admissible Semantics

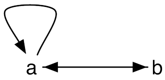
Example

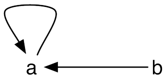




SE wrt. Admissible Semantics

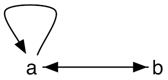
Example

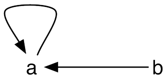




SE wrt. Admissible Semantics

Example





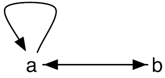
a-kernel

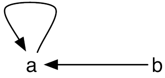
For an AF $F = (A, R)$ we define the a-kernel of F as $F^{ak} = (A, R^{ak})$ where

$$R^{ak} = R \setminus \{(a, b) \mid a \neq b, (a, a) \in R, \{(b, a), (b, b)\} \cap R \neq \emptyset\}.$$

SE wrt. Admissible Semantics

Example





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SE wrt. Admissible Semantics

- For any AF F and G, $F^{ak} = G^{ak} \implies F^{sk} = G^{sk}$
- For any AF F, $\sigma(F) = \sigma(F^{ak})$ for $\sigma \in \{adm, pref, ideal, semi, eager\}$
- If $F^{ak} = G^{ak}$, then $(F \cup H)^{ak} = (G \cup H)^{ak}$ for each AF H
- For any AFs F and G: $F^{ak} = G^{ak}$ iff $F \equiv_s^{\sigma} G$ for $\sigma \in \{adm, pref, ideal, semi, eager\}$

SE wrt. Grounded Semantics

Example

SE wrt. Grounded Semantics

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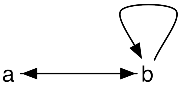
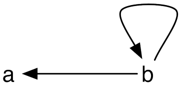




SE wrt. Grounded Semantics

Example

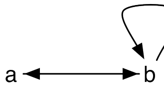
g-kernel

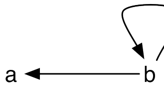
For an AF $F = (A, R)$ we define the g-kernel of F as $F^{gk} = (A, R^{gk})$ where

$$R^{gk} = R \setminus \{(a,b) \mid a \neq b, (b,b) \in R, \{(a,a),(b,a)\} \cap R \neq \emptyset\}.$$

SE wrt. Grounded Semantics

Example





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SE wrt. Grounded Semantics

- For any AF F, $ground(F) = ground(F^{gk})$
- Let F and G be AFs, s.t. $F^{gk} = G^{gk}$. Then, $(F \cup H)^{gk} = (G \cup H)^{gk}$ for each AF H
- For any AFs F and G: $F^{gk} = G^{gk}$ iff $F \equiv_s^{ground} G$

SE wrt. Complete Semantics

Example

c-kernel

For an AF $F = (A, R)$ we define the c-kernel of F as $F^{ck} = (A, R^{gk})$ where

$$R^{ck} = R \setminus \{(a, b) \mid a \neq b, (a, a), (b, b) \in R\}.$$

SE wrt. Complete Semantics

c-kernel

For an AF $F = (A, R)$ we define the c-kernel of F as $F^{ck} = (A, R^{gk})$ where

$$R^{ck} = R \setminus \{(a,b) \mid a \neq b, (a,a), (b,b) \in R\}.$$

SE wrt. Complete Semantics

- For any AFs F and G, $F^{ck} = G^{ck} \implies F^{\tau} = G^{\tau}$ for $\tau \in \{sk, ak, gk\}$
- Let F and G be AFs, s.t. $F^{ck} = G^{ck}$ iff jointly $F^{ak} = G^{ak}$ and $F^{gk} = G^{gk}$
- For any AF F, $comp(F) = comp(F^{ck})$
- Let F and G be AFs, s.t. $F^{ck} = G^{ck}$. Then, $(F \cup H)^{ck} = (G \cup H)^{ck}$ for each AF H
- For any AFs F and G: $F^{ck} = G^{ck}$ iff $F \equiv_s^{comp} G$

SE and Self-Loops

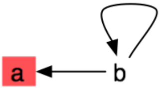
Self-Loop Free AFs

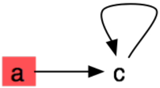
For any self-loop free AF F,

$$F = F^{sk} = F^{ak} = F^{ck} = F^{gk}$$

Outline

SE wrt. *naive* Semantics





- $naive(F) = naive(G) = \{\{a\}\}$

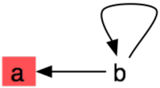
SE wrt. *naive* Semantics

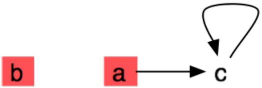
- $naive(F \cup H) = naive(G \cup H) = \{\{d\}, \{a, e\}\}$

SE wrt. *naive* Semantics

- $naive(F \cup H) = naive(F) = \{\{a\}\}$ but
- $naive(G \cup H) = \{\{a, b\}\}$.

SE wrt. *naive* Semantics





- $naive(F \cup H) = naive(F) = \{\{a\}\}$ but
- $naive(G \cup H) = \{\{a, b\}\}$.

Theorem ([Gaggl and Woltran, 2013])

The following statements are equivalent:

1. $F \equiv_s^{naive} G$;
2. $naive(F) = naive(G)$ and $A(F) = A(G)$;
3. $cf(F) = cf(G)$ and $A(F) = A(G)$.

Strong Equivalence wrt. *cf2*

Theorem ([Gaggl and Woltran, 2013])

For any AFs F and G, $F \equiv_s^{cf2} G$ iff $F = G$.

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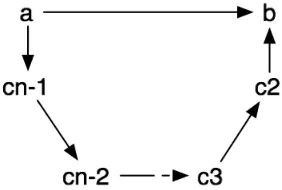

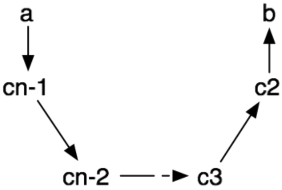

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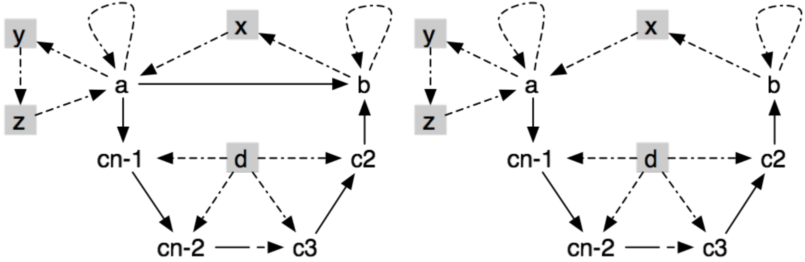

$$H = (A \cup \{d, x, y, z\}, \{(a,a),(b,b),(b,x),(x,a),(a,y),(y,z),(z,a), (d,c) \mid c \in A \setminus \{a,b\}\}).$$

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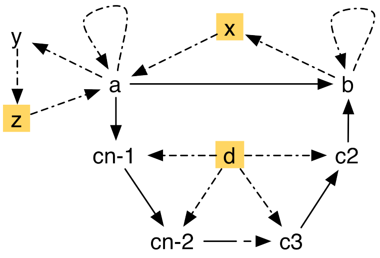

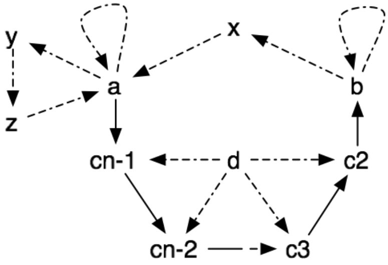

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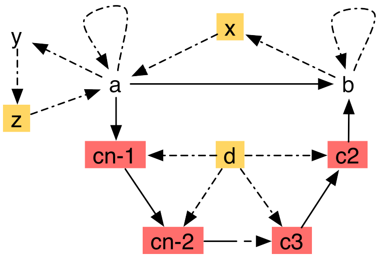

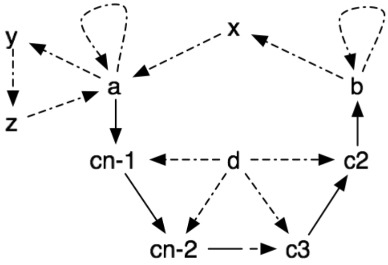

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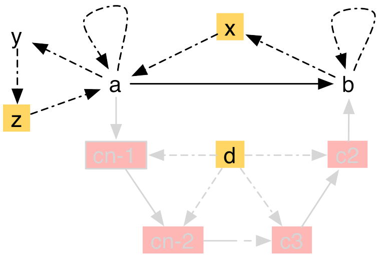

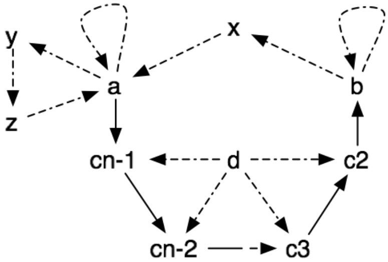

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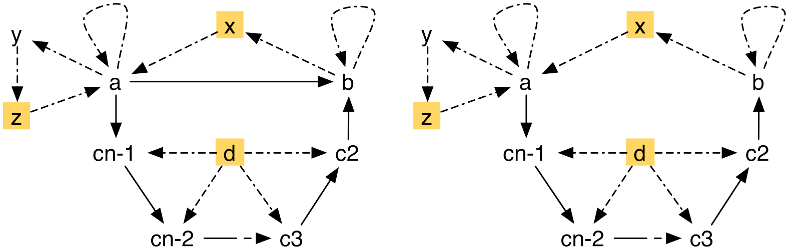

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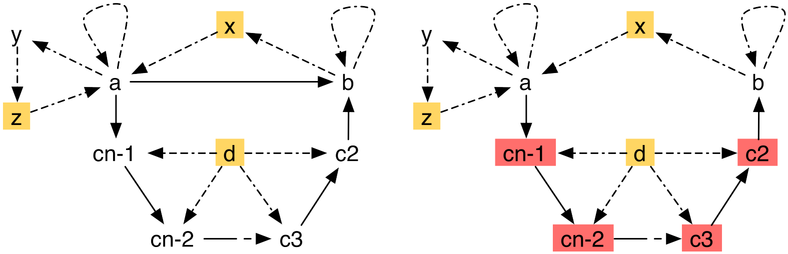

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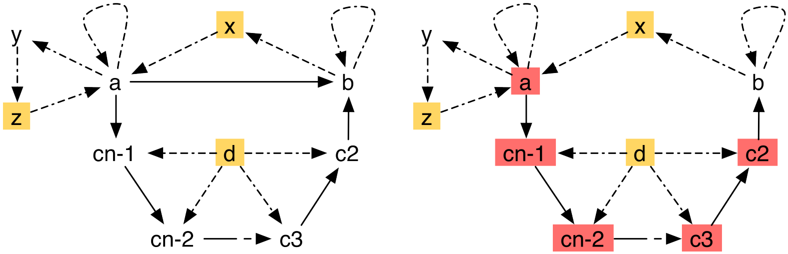

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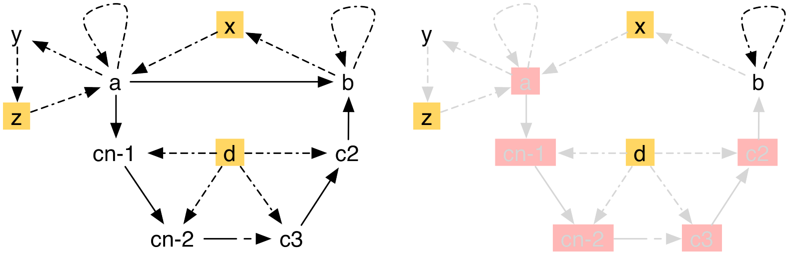

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SE wrt. *cf2*

Theorem

For any AFs F and G, $F \equiv_s^{cf2} G$ iff $F = G$.

- No matter which AFs $F \neq G$, one can always construct an H s.t. $cf2(F \cup H) \neq cf2(G \cup H)$;

SE wrt. *cf2*

Theorem

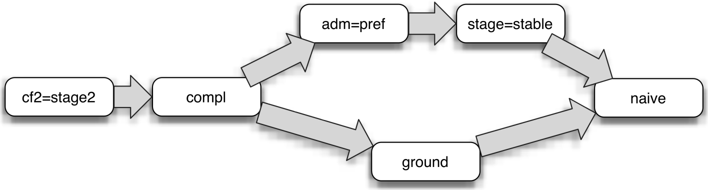
For any AFs F and G, $F \equiv_s^{cf2} G$ iff $F = G$.

- No matter which AFs $F \neq G$, one can always construct an H s.t. $cf2(F \cup H) \neq cf2(G \cup H)$;

Succinctness Property [Gaggl and Woltran, 2013]

An argumentation semantics σ satisfies the succinctness property or is maximal succinct iff no AF contains a redundant attack wrt. σ.

Comparing Semantics wrt. SE



Outline

Expansion of an AF

[Baumann and Brewka, 2013, Baumann, 2012]

Back to H ...

- H might be of certain nature, s.t. $F \cup H$ can be characterized,
- which in turn yields different (strong) equivalences.

Expansion

An AF F^* is an expansion of AF $F = (A, R)$ (for short $F \preceq_E F^*$), iff $F^* = (A \cup A^*, R \cup R^*)$ where $A \cap A^* = R \cap R^* = \emptyset$. An expansion is

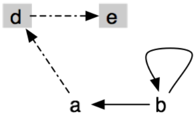
1. normal ($F \preceq_N F^*$), iff $\forall a, b\ ((a, b) \in R^* \rightarrow a \in A^* \vee b \in A^*)$,
2. strong ($F \preceq_S F^*$), iff $F \preceq_N F^*$ and $\forall a, b\ ((a, b) \in R^* \rightarrow \neg(a \in A \wedge b \in A^*))$,
3. weak ($F \preceq_W F^*$), iff $F \preceq_N F^*$ and $\forall a, b\ ((a, b) \in R^* \rightarrow \neg(a \in A^* \wedge b \in A))$,
4. local ($F \preceq_L F^*$), iff $A^* = \emptyset$.

Expansions of an AF

Expansion

An AF F^* is an expansion of AF $F = (A, R)$ (for short $F \preceq_E F^*$), iff $F^* = (A \cup A^*, R \cup R^*)$ where $A \cap A^* = R \cap R^* = \emptyset$. An expansion is

1. normal ($F \preceq_N F^*$), iff $\forall a, b\ ((a, b) \in R^* \rightarrow a \in A^* \vee b \in A^*)$,
2. strong ($F \preceq_S F^*$), iff $F \preceq_N F^*$ and $\forall a, b\ ((a, b) \in R^* \rightarrow \neg(a \in A \wedge b \in A^*))$,
3. weak ($F \preceq_W F^*$), iff $F \preceq_N F^*$ and $\forall a, b\ ((a, b) \in R^* \rightarrow \neg(a \in A^* \wedge b \in A))$,
4. local ($F \preceq_L F^*$), iff $A^* = \emptyset$.



- F^* is a weak expansion
- F^* is NOT strong or local

Notions of Equivalence

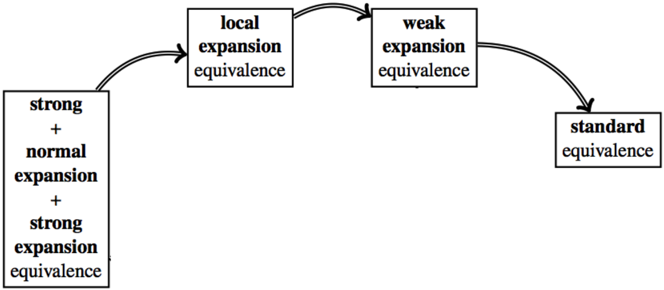
Equivalence relations are now developed wrt. the type of expansion.

Notions of Equivalence

Given a semantic σ. Two AFs F and G are

- normal expansion equivalent wrt. σ ($F \equiv_N^\sigma G$) iff for each AF H, s.t. $F \preceq_N F \cup H$ and $G \preceq_N G \cup H$, $F \cup H \equiv^\sigma G \cup H$ holds,
- strong expansion equivalent wrt. σ ($F \equiv_S^\sigma G$) iff for each AF H, s.t. $F \preceq_S F \cup H$ and $G \preceq_S G \cup H$, $F \cup H \equiv^\sigma G \cup H$ holds,
- weak expansion equivalent wrt. σ ($F \equiv_W^\sigma G$) iff for each AF H, s.t. $F \preceq_W F \cup H$ and $G \preceq_W G \cup H$, $F \equiv^\sigma G \cup H$ holds,
- local expansion equivalent wrt. σ ($F \equiv_L^\sigma G$) iff for each AF G, s.t. $A(H) \subseteq A(F \cup G)$, $F \cup H \equiv^\sigma G \cup H$ holds.

Relations for Stable Semantics



Outline

Summary

- Most reasoning problems for AFs are intractable
- Translations Tr for semantics $\sigma \Rightarrow \sigma'$ s.t., for each AF F, $\sigma(F) = \sigma'(Tr(F))$
- We identified kernels for stable, admissible ($pref, ideal, semi, eager$), complete and grounded semantics
- We provide characterizations for strong equivalence wrt. stage, naive and *cf2* semantics.
- *cf2* semantics is the only one where no redundant attacks exist.
- *cf2* semantics treats self-loops in a more sensitive way than other semantics.
- More fine grained characterization of equivalence wrt. expansions

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