Technische Universität Dresden

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Formal Concept Analysis

Exercise Sheet 1, Winter Semester 2015/16

1 Set Theory

Exercise 1 (a piece of recapitulation)

Given the following hints and the universe $M := \{1, 2, 3, 4, 5, 6, 7, 8\}$, compute the sets A, B, C:

(*a*) $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

(*b*) $B \cup C = \{1, 2, 4, 6, 8\}$

(*c*) $A \cup C = \{1, 2, 3, 4, 5, 7, 8\}$

(*d*) $A \cap B = \{2\}$

(*e*) $B \cap C = \{2, 4, 8\}$

(*f*) $A \cap C = \{2\}$

2 Logic

Exercise 2 (repetition first-order logic)

Formalize the following statements for natural numbers a, b, c, using only multiplication ("$\cdot$"), equality ("$=$") and natural numbers ("0","1","2",...) besides the usual logical symbols ("$\neg$", "$\wedge$", "$\vee$", "$\rightarrow$", "$\leftrightarrow$", "$\forall$", "$\exists$", variables and parentheses):

(*i*) a divides b.

(*ii*) a is odd.

(*iii*) a is common divisor of b and c

(*iv*) a is the gcd of b and c.

(*v*) a is a square number.

(*vi*) a is a prime number.

3 Derivation Operators and Formal Concepts

Exercise 3 (line diagram)

a) Recall: how is the derivation operator $(\cdot)'$ defined?

b) Let $\mathbb{K} = (G, M, I)$ be a formal context and let $A, B \subseteq G$. Prove the following statements:

1. $A \subseteq B$ implies $B' \subseteq A'$
2. $A \subseteq A''$
3. $A' = A'''$
4. For $C \in G$ and $D \in M$ holds: (C, D) is a formal concept if and only if there is some $E \subseteq G$ such that $C = E''$ and $D = E'$.

4 Formal Concept Analysis

Exercise 4 (Formal Context)

Regard the following formal context $\mathbb{K}$, given as a cross table:

	needs water to live	lives in water	lives on land	needs chlorophyll to produce food	two seed leaves	one seed leaf	can move around	has limbs	suckles its offspring
Leech	x	x					x		
Bream	x	x					x	x	
Frog	x	x	x				x	x	x
Spike-Weed	x	x		x		x			
Reed	x	x	x	x		x			
Bean	x		x	x	x				
Maize	x		x	x		x			

a) Specify the following sets:

- (i) $\{\text{Bean}\}'$
- (ii) $\{\text{lives on land}\}'$
- (iii) $\{\text{two seed leaves}\}''$
- (iv) $\{\text{Frog, Maize}\}'$
- (v) $\{\text{needs chlorophyll to produce food, can move around}\}'$
- (vi) $\{\text{lives in water, lives on land}\}'$
- (vii) $\{\text{needs chlorophyll to produce food, can move around}\}''$

b) Extend $\mathbb{K}$ with both an object and an attribute.