TECHNISCHE UNIVERSITÄT DRESDEN

KNOWLEDGE GRAPHS

Lecture 8: Expressive Power and Complexity of SPARQL

Markus Krötzsch

Knowledge-Based Systems

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Review

SPARQL is a feature-rich query language:

- Basic graph patterns (conjunctions of triple patterns)
- Property path patterns
- Filters
- Union, Optional, Minus,
- Subqueries, Values, Bind
- Solution set modifiers
- Aggregates

Semantics of each feature is defined by specific algebra operators

Translating SPARQL to nested algebra expressions is mostly straightforward, but involves some special cases that need to be observed (e.g., how Filter expressions in optional parts are treated).

Complexity of SPARQL

Review: Complexity of BGP matching

We had already observed the following basic result:

Theorem 4.12: Determining if a BGP has solution mappings over a graph is NP-complete.

Our proof was by reduction from graph 3-colourability, a well-known NP-hard problem:

The problem of graph 3-colourability (**3Col**) is defined as follows:
Given: An undirected graph G
Question: Can the vertices of G be assigned colours red, green and blue so that no two adjacent vertices have the same colour?

Proof idea: Encode the given graph as a BPG and use a three-vertex colouring template as RDF graph to match it to.

NP-hardness another way

A typical NP-complete problem is satisfiability of propositional logic formulae:

The problem of propositional logic satisfiability (**SAT**) is defined as follows:
Given: An propositional logic formula φ
Question: Is it possible to assign truth values to propositional variables in φ such that the formula evaluates to true?

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Exercise: Give a direct reduction from **SAT** to SPARQL query answering, without using BGPs.

This shows (in another way) that SPARQL query answering is NP-hard. However, it is actually harder than that.

Beyond NP

In complexity theory, space is usually more powerful than time
(intuition: space can be reused; time, alas, cannot)

Definition 8.1: Let $f : \mathbb{N} \to \mathbb{R}^+$ be a function. A Turing machine $\mathcal{M}$ is $O(f)$-space bounded if there is a function $g \in O(f)$ such that $\mathcal{M}$ halts on every input $w \in \Sigma^*$ using $\leq g(|w|)$ cells on its tapes.

DSpace$(f(n))$ is the class of all languages **L** for which there is an $O(f(n))$-space bounded Turing machine deciding **L**.

The class PSpace of languages decidable in polynomial space is defined as PSpace $= \bigcup_{d \geq 1}$ DSpace(n^d).

Space restrictions can also be used for non-deterministic Turing machines, but by Savitch's Theorem, this does not give additional expressive power: PSpace = NPSpace

Indeed, it is known that P $\subseteq$ NP $\subseteq$ PSpace (and all inclusions are believed to be strict, though this remains unproven)

Quantified Boolean Formulae

A QBF is a formula of the following form:

$$\mathsf{Q}_1 X_1.\mathsf{Q}_2 X_2.\cdots\mathsf{Q}_\ell X_\ell.\varphi[X_1,\ldots,X_\ell]$$

where $\mathsf{Q}_i \in \{\exists, \forall\}$ are quantifiers, X_i are propositional logic variables, and φ is a propositional logic formula with variables $X_1,\ldots,X_\ell$ and constants $\top$ (true) and $\bot$ (false)

Semantics:

- Propositional formulae without variables (only constants $\top$ and $\bot$) are evaluated as usual
- $\exists X.\varphi[X]$ is true if either $\varphi[X/\top]$ or $\varphi[X/\bot]$ are true
- $\forall X.\varphi[X]$ is true if both $\varphi[X/\top]$ and $\varphi[X/\bot]$ are true

  (where $\varphi[X/\top]$ is "φ with X replaced by $\top$, and similar for $\bot$)

Hardness of QBF Evaluation

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Given: A quantified boolean formula φ

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This is a rather difficult question:

Example 8.2: A propositional formula φ with propositions $p_1, \ldots, p_n$ is satisfiable if $\exists p_1 \ldots \exists p_n.\varphi$ is a true QBF, i.e., **SAT** reduces to **TrueQBF** (so it is NP-hard).

The QBF φ is a tautology if $\forall p_1 \ldots \forall p_n.\varphi$ is a true QBF, i.e., tautology checking reduces to **TrueQBF** (so it is coNP-hard).

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In fact, it is known that **TrueQBF** is harder than both NP and coNP:

Theorem 8.3: **TrueQBF** is PSpace-complete.

(without proof; see course “Complexity Theory”)

Universal quantifiers in SPARQL

To show NP-hardness, we used the fact that SPARQL can naturally express existential quantifiers, since we always ask “does a match for this query exist”?

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Can we also express universal quantifiers? — Yes:

Example 8.4: In Wikidata, find bands all of whose (known) members are female.

```
SELECT ?band
WHERE {
 ?band wdt:P31 wd:Q215380 . # ?band instance of: band
 ?band wdt:P527 [] . # ?band has part: [] (at least one known member)
 FILTER NOT EXISTS {
   ?band wdt:P527 ?member . # ?band has part: ?member
   FILTER NOT EXISTS {
     ?member wdt:P21 wd:Q6581072 # ?member sex or gender: female
   }
 }
}
```

SPARQL is PSpace-hard

The PSpace-hardness of **TrueQBF** + the encoding universal quantifiers yield:

Theorem 8.5: Deciding whether a SPARQL query has any results is PSpace-hard, even over an empty RDF graph.

Proof: We reduce QBF formulae to SPARQL queries.

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2. Replace the innermost boolean formula (φ or $\neg\varphi$) by an expression `FILTER` $(\hat{\varphi})$ where $\hat{\varphi}$ is $(\neg)\varphi$ written using SPARQL Boolean functions $\&\&$, ||, and !, and with each propositional variable X_i replaced by a unique SPARQL variable `?Xi`.

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From the resulting SPARQL expression `P`, create the query:

```
SELECT * WHERE { VALUES ?x {"QBF is true!"} P }
```

SPARQL is PSpace-hard (2)

It is not hard to see that this transformation works as desired: the resulting query has a solution mapping $\{x \mapsto$ `"QBF is true!"`$\}$ if and only if the QBF is true.

Example 8.6: Consider the QBF $\forall p.\exists q.((\neg p \wedge q) \vee (p \wedge \neg q))$. Eliminating $\forall$ yields $\neg\exists p.\neg\exists q.((\neg p \wedge q) \vee (p \wedge \neg q))$. We then obtain the following SPARQL query:

```
SELECT * WHERE {
 VALUES ?x {"QBF is true!"}
 FILTER NOT EXISTS { VALUES ?p {true false}
    FILTER NOT EXISTS { VALUES ?q {true false}
       FILTER ( (! ?p && ?q) || (?p && ! ?q) )
    }
 }
}
```

Is SPARQL practical?

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Is complexity theory useless?

No, but we should measure more carefully:

- Our proofs (for NP and PSpace) turn hard problems into hard queries
- We hardly need RDF data at all

In practice, databases grow very big, while queries are rather limited!

(Wikidata has over 5 Billion triples; typical Wikidata query have less than 100 triple patterns [Malyshev et al., ISWC 2018])

More fine-grained complexity measures

Combined Complexity
Input: Query Q and RDF graph G
Output: Does Q have answers over G?

$\leadsto$ estimates complexity in terms of overall input size
$\leadsto$ "2KB query/2TB database" = "2TB query/2KB database"

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$\leadsto$ we can also fix the database and vary the query:

Query Complexity
Input: SPARQL query Q
Output: Does Q have answers over G? (for fixed RDF graph G)

Below P

Our previous proofs show high query complexity (hence also high combined complexity). For data complexity, we get much lower complexities, starting below polynomial time.

Definition 8.7: The class NL of languages decidable in logarithmic space on a non-deterministic Turing machine is defined as NL $=$ NSpace$(\log(n))$.

Note: When restricting Turing machines to use less than linear space, we need to provide them with a separate read-only input tape that is not counted (since the input of length n cannot fit into $\log(n)$ space itself).

Intuition: The memory of a logspace-bounded Turing machine (deterministic or not) is just enough for the following:

- Store a fixed number of binary counters (with at most polynomial value)
- Store a fixed number of pointers to positions in the input
- Compare the values of counters and target symbols of pointers

It is known that NL $\subseteq$ P $\subseteq$ NP (and all inclusions are believed to be strict, though this remains unproven)

Data complexity of SPARQL

The problem of directed graph reachability (also known as s-t-reachability) is defined as follows:

Given: A directed graph G and two vertices s and t

Question: Is there a directed path from s to t?

This can be solved in NL:

- Starting from s, non-deterministically move to a successor vertex
- Terminate when moving to t (success) or after making more moves than vertices in the graph (failure)

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Directed graph reachability is furthermore known to be NL-hard, so we get:

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Proof: Directed graph reachability is easily reduced: encode graph in RDF, and use a single property path pattern with * to check reachability. □

Upper bounds

Important note: All of our results so far were lower bounds, showing that SPARQL is at least as hard as the given class. We have not shown that SPARQL queries can actually be answered in the given bounds.[1]

[1] We have not even shown that SPARQL query answers are computable at all. SQL query answers, e.g., are not, if all SQL features are allowed.

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How to obtain upper bounds?

- Give an algorithm
- Show that it can run within the required bounds (with respect to query size and/or data size)

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Problem: SPARQL has a large number of features that an algorithm would need to consider, making algorithms rather complex and harder to verify

$\leadsto$ sketch algorithms for basic cases only

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Answering queries in PSpace

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Algorithm sketch:

- Iterate over all possible variable and bnode bindings, storing them one by one (possible in polynomial space)
- Verify query conditions for the given binding (possible in polynomial space for most features, e.g., triple patterns, property path patterns, filters, union, minus, . . .)

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Where this sketch is lacking:

- We should check complexity of all filter conditions and functions
- We did not clarify how to handle subqueries and aggregates
- Result values can become exponentially large (e.g., by repeated string doubling using `BIND`), so a smarter representation of values has to be used

Answering queries in NL for data

We can use the same approach for worst-case optimal query answering with respect to the size of the RDF graph (data complexity):

Algorithm sketch:

- Iterate over all possible variable and bnode bindings, storing one at a time
- Verify query conditions for the given binding

$\leadsto$ If the query is fixed, the bindings can be stored using a fixed number of pointers.

$\leadsto$ For most operations, it is again clear that they are possible to verify in NL

This includes many numeric aggregates and arithmetic operations.

Again, we omit many details here that would need careful discussion.

Note: In terms of the size of the data, values can not be exponentially but merely polynomially large, since the query is constant now; but one still needs to explain how to represent this.

Outlook

SPARQL: Outlook

A number of SPARQL features have not been discussed:

- Graphs: SPARQL supports querying RDF datasets with multiple graphs, and queries can retrieve graph names as variable bindings
- Updates: SPARQL has a set of features for inserting and deleting data

Example 8.9: The following query replaces all uses of the `hasSister` property with a different encoding of the same information:

```
DELETE { ?person eg:hasSister ?sister }
INSERT {
 ?person eg:hasSibling ?sister .
 ?sister eg:sex eg:female .
}
WHERE { ?person eg:hasSister ?sister }
```

- Result formats: SPARQL has several encodings for sending results, and it can also encode results as RDF graphs (**CONSTRUCT**).
- Federated queries: SPARQL can get sub-query results from other SPARQL services

Teaching evaluation

Summary

SPARQL is PSpace-complete for query and combined complexity[1]

SPARQL is NL-complete for data complexity, hence practically tractable and well parallelisable[1]

SPARQL expressivity is still limited, partly by design.

What's next?

- Property graph: another popular graph data model
- The Cypher query language
- Quality assurance in knowledge graphs

[1] The matching upper bound has not been proven with the full set of features.