

Algorithmic Game Theory

Summer Term 2024

exercises 1

15–19/04/2024

Problem 1.

Consider the games **the battle of the partners** as well as **the prisoner's dilemma** with the payoff tables displayed below. Although you already know the answers to the questions below from the lecture, work closely with the central definitions to answer them again.

- What strategy profiles provide Nash equilibria in both games?
- Is there a difference between the equilibria in both games?
- Are the equilibria Pareto optimal in both games?
- Are there dominant strategies in both games?

Battle of the Partners:

<i>(Helena, George)</i>	Soccer	Concert
Soccer	(10,1)	(0,0)
Concert	(0,0)	(0,10)

Prisoner's Dilemma:

<i>(Smith, Wesson)</i>	Confession	Silence
Confession	(-4,-4)	(0,-10)
Silence	(-10,0)	(-2,-2)

Problem 2.

Consider the following two exercises.

- Imagine that you are a fully rational decision maker facing a (one-shot) prisoner's dilemma. Explain why the outcome of the game will not be affected if you are allowed to meet and discuss with the other player before you make your move.
- By definition, a non-cooperative game is a game in which the players are not able to form binding agreements. Why can we not just say that a non-cooperative game is a game in which the players do not actually cooperate?

Problem 3.

Consider the following 3×3 game.

<i>(Player₁, Player₂)</i>	l	c	r
T	(1,0)	(3,1)	(1,1)
M	(1,1)	(3,0)	(0,1)
B	(2,2)	(3,3)	(0,2)

- (a) Identify all pairs of strategies where one strategy weakly dominates the other.
- (b) Assume you are allowed to remove a weakly dominated strategy of some player. Do so, and repeat this process (of iterated elimination of weakly dominated strategies) until you find a single strategy pair of the original game.
- (c) Find such an iterated elimination of weakly dominated strategies that results in a strategy pair other than the one found in (b), where both strategies, and the payoffs to the players, are different.
- (d) What are the equilibria (in pure strategies) of the game?

Problem 4.

Consider the following three-player game in strategic form.

$(Player_1, Player_2, Player_3(L))$	l	r
T	(3,4,4)	(1,3,3)
B	(8,1,4)	(2,0,6)

$(Player_1, Player_2, Player_3(R))$	l	r
T	(4,0,5)	(0,1,6)
B	(5,1,3)	(1,2,5)

- (a) Identify all pairs of strategies where one strategy strictly, or weakly, dominates the other.
- (b) Apply iterated elimination of strictly dominated strategies to this game. What are the Nash equilibria of the game?