Exercise Sheet 5: Treewidth and Hypertreewidth

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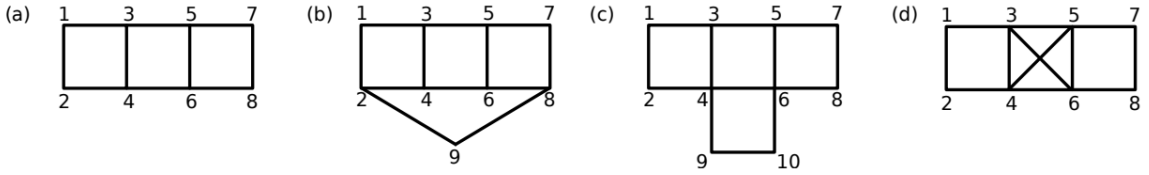
Database Theory, 2020-06-01, Summer Term 2021

Exercise 5.1. Construct the query hypergraph and the primal graph for the following queries:

1. $\exists x, y, z, u, v.\ r(x, y, z, u) \wedge s(z, u, v)$
2. $\exists x, y, z, u, v.\ a(x, y) \wedge b(y, z) \wedge c(z, u) \wedge d(u, v) \wedge e(v, z) \wedge f(z, x) \wedge d(x, u) \wedge d(u, y)$

Exercise 5.2. Determine the treewidth of each of the following graphs and provide a suitable tree decomposition.



Exercise 5.3. Show that a clique (fully connected graph) of size n has treewidth $n - 1$.

Exercise 5.4. Decide whether the following claims are true or false. Explain your answer.

1. Deleting an edge from a graph may make the treewidth smaller but never larger.
2. Deleting a vertex from a graph (and removing all of its edges) may make the treewidth smaller but never larger.
3. Deleting a hyperedge from a hypergraph may make the hypertree width smaller but never larger.
4. Deleting a vertex from a hypergraph (and contracting all of its edges) may make the hypertree width smaller but never larger.

Exercise 5.5. The following BCQ corresponds to graph (a) in Exercise 5.2:

$$\exists x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8.\ r(x_1, x_2) \wedge r(x_1, x_3) \wedge r(x_2, x_4) \wedge r(x_3, x_4) \wedge r(x_3, x_5) \wedge r(x_4, x_6) \wedge r(x_5, x_6) \wedge r(x_5, x_7) \wedge r(x_6, x_8) \wedge r(x_7, x_8)$$

According to the logical characterisation from the lecture, this query can be expressed in the $\exists$-$\wedge$-fragment of FO using only treewidth$+1$ variables. Find such a formula.