EXERCISE 5

Science of Computational Logic

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Problem 5.1

Show that first-order logic is monotonic.

Problem 5.2

Show that reasoning with CWA is non-monotonic.

Problem 5.3

Consider the language $\mathcal{L}(\mathcal{R}, \mathcal{F}, \mathcal{V})$, with $\mathcal{R} = \{p/0, q/0\}$.
Given the set of formulas $\mathcal{S} = \{p \leftarrow \neg q, q \leftarrow \neg p\}$

Compute $C_{CWA}(\mathcal{S})$.

Problem 5.4

Prove that the closed world assumption eliminated non-least Herbrand models:

If F is a formula and I is a non-least Herbrand model I of F , then $I \not\models C_{CWA(F)}$.

Problem 5.5

Proof the following proposition:
Let $\mathcal{F}$ be a satisfiable set of Skolem formulas. Then it holds:

$C_{CWA}(\mathcal{F})$ is satisfiable $\Leftrightarrow$ $\mathcal{F}$ admits a least Herbrand model.

Problem 5.6

Reconsider the theorem from the lectures proved in the preceding problem.

1. Show that the condition that $\mathcal{F}$ a set of formulas in Skolem normal form is necessary for $\Rightarrow$ -direction.
2. Show for the $\Leftarrow$ -direction that without the condition that $\mathcal{F}$ a set of formulas in Skolem normal form the existence of a least Herbrand model of $\mathcal{F}$ does not entail the existence of a Herbrand model of $C_{CWA}(\mathcal{F})$.