On the Computational Complexity of Naive-based Semantics for Abstract Dialectical Frameworks

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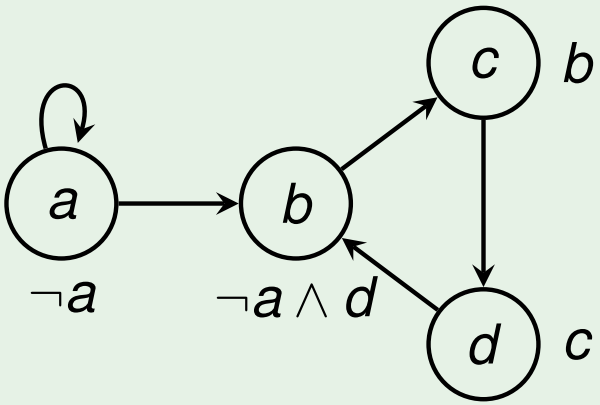
Motivation

- Abstract dialectical frameworks (ADFs): a powerful generalisation of Dung's abstract argumentation frameworks
- ADFs allow to model argumentation scenarios s.t. that ADF semantics provide interpretations of scenarios
- Naive-based semantics are built upon the fundamental concept of conflict-freeness
- Exhaustive analysis of computational complexity of naive-based semantics
- Interesting results: some involve little-known classes of so-called Boolean hierarchy (another hierarchy in between classes of polynomial hierarchy)
- Credulous and sceptical entailment: different complexity depending on whether we check for truth or falsity of statement

ADFs and Semantics

Example



$$cfi(D) = \{\{a \mapsto \mathbf{u}, b \mapsto \mathbf{u}, c \mapsto \mathbf{u}, d \mapsto \mathbf{u}\},$$
$$\{a \mapsto \mathbf{u}, b \mapsto \mathbf{t}, c \mapsto \mathbf{u}, d \mapsto \mathbf{u}\},$$
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$$\{a \mapsto \mathbf{u}, b \mapsto \mathbf{u}, c \mapsto \mathbf{u}, d \mapsto \mathbf{t}\},$$
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$$\{a \mapsto \mathbf{u}, b \mapsto \mathbf{u}, c \mapsto \mathbf{t}, d \mapsto \mathbf{t}\},$$
$$\{a \mapsto \mathbf{u}, b \mapsto \mathbf{t}, c \mapsto \mathbf{t}, d \mapsto \mathbf{t}\},$$
$$\{a \mapsto \mathbf{u}, b \mapsto \mathbf{f}, c \mapsto \mathbf{f}, d \mapsto \mathbf{f}\}\}$$

$$nai(D) = \{\{a \mapsto \mathbf{u}, b \mapsto \mathbf{t}, c \mapsto \mathbf{t}, d \mapsto \mathbf{t}\},$$
$$\{a \mapsto \mathbf{u}, b \mapsto \mathbf{f}, c \mapsto \mathbf{f}, d \mapsto \mathbf{f}\}\}$$

Definition

Let $D = (S, L, C)$ be an ADF. A three-valued interpretation $v : S \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$ is

- conflict-free, i.e. $v \in cfi(D)$, iff for all $s \in S$ we have:
  - $v(s) = \mathbf{t}$ implies that φ_s^v is satisfiable,
  - $v(s) = \mathbf{f}$ implies that φ_s^v is unsatisfiable;
- naive, i.e. $v \in nai(D)$, iff v is $\leq_i$-maximal conflict-free;

Main Results

	cfi	*nai*	*stg*	nai_2
Ver_σ	DP-c	Π_2^P-c	Π_2^P-c	Π_2^P-c
Exists_σ	coDP-c	coDP-c	coDP-c	coDP-c
$\text{Cred}_\sigma^{\mathbf{t}}$	NP-c	NP-c	Σ_3^P-c	Σ_3^P-c
$\text{Cred}_\sigma^{\mathbf{f}}$	Σ_2^P-c	Σ_2^P-c	Σ_3^P-c	Σ_3^P-c
$\text{Scep}_\sigma^{\mathbf{t}}$	trivial	Π_2^P-c	Π_3^P-c	Π_3^P-c
$\text{Scep}_\sigma^{\mathbf{f}}$	trivial	Π_3^P-c	Π_3^P-c	Π_3^P-c

Decision Problems

Let $D = (S, L, C)$ be an ADF, σ a semantics.

- Ver_σ: given $v : S \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$, is $v \in \sigma(D)$?
- Exists_σ: does there exist a non-trivial interpretation $v \in \sigma(D)$?
- $\text{Cred}_\sigma^{\mathbf{t}}$ / $\text{Cred}_\sigma^{\mathbf{f}}$: $s \in S$, does there exist an interpretation $v \in \sigma(D)$ with $v(s) = \mathbf{t}$ / $v(s) = \mathbf{f}$?
- $\text{Scep}_\sigma^{\mathbf{t}}$ / $\text{Scep}_\sigma^{\mathbf{f}}$: $s \in S$, is $v(s) = \mathbf{t}$ / $v(s) = \mathbf{f}$ for all $v \in \sigma(D)$?