Foundations of Semantic Web Technologies

Solutions for Tutorial 4: Tableau

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Solution (4.1)**.**

(a) NNF($\neg(A \sqcap \forall r.B)$) $\Rightarrow \neg A \sqcup \exists r.(\neg B)$

(b) NNF($\neg\forall r.\exists s.(\neg B \sqcup \exists r.A)$) $\Rightarrow \exists r.\forall s.(B \sqcap \forall r.(\neg A))$

(c) NNF($\neg((\neg A \sqcap \exists r.\top) \sqcup \geqslant 3\, s.(A \sqcup \neg B))$) $\Rightarrow (\neg(\neg A \sqcap \exists r.\top) \sqcap \neg(\geqslant 3\, s.(A \sqcup \neg B))) \Rightarrow ((A \sqcup \forall r.\bot) \sqcap \leqslant 2\, s.(A \sqcup \neg B))$

Solution (4.2)**.**
We want to test, whether A is a subset of B in all models of the knowledge base. Since we can not construct all models, we reformulate the (subsumption) problem and test now for the complement $A \sqcap \neg B$, thereby requesting a model where an individual is in A but not in B. If such a model can be determined, the subsumption does NOT hold, otherwise we conclude that $A \sqsubseteq B$ is entailed by the knowledge base.
$C_{\mathcal{T}} = (C \sqcup B) \sqcap (\neg A \sqcup \neg C \sqcup \bot)$ simplified $C_{\mathcal{T}} = (C \sqcup B) \sqcap (\neg A \sqcup \neg C)$. The tableaux is initialized with node v_0, labeled with A and $\neg B$.
$v_0 : \{A, \neg B, C_{\mathcal{T}}, C \sqcup B, \neg A \sqcup \neg C\}$

We try it with the first disjunct of each disjunction:
$v_0 : \{A, \neg B, C_{\mathcal{T}}, C \sqcup B, \neg A \sqcup \neg C, C, \neg A\}$

Which yields the contradiction A and $\neg A$, and we try $\neg C$ instead of $\neg A$:
$v_0 : \{A, \neg B, C_{\mathcal{T}}, C \sqcup B, \neg A \sqcup \neg C, C, \neg C\}$

But again, this yields the contradiction C and $\neg C$, which forces us to consider the first disjunction again and choose B.
$v_0 : \{A, \neg B, C_{\mathcal{T}}, C \sqcup B, \neg A \sqcup B\}$

This also yields a contradiction and we can not apply any rule further, which leaves us with a closed tableaux. Consequently, the subsumption $A \sqsubseteq B$ holds,i.e. it is entailed by the TBox.

Solution (4.3)**.**
Since we deal with existential quantifiers now, we need to consider blocking. And since also inverse roles are used, we have to use equality blocking.

$$C_{\mathcal{T}} = (\neg A \sqcup \exists r.A) \sqcap (\neg B \sqcup \exists r^{-}.C) \sqcap (\neg C \sqcup \forall r.\forall r.B)$$

The tableau is initialized with node v_0 labeled by $\{A \sqcap \forall r.B\}$ and $C_{\mathcal{T}}$. Applying the $\sqcap$-Rule yields:
$v_0 : \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^{-}.C, \neg C \sqcup \forall r.\forall r.B\}$

Considering the disjunctions, we choose $\exists r.A$ from the first one since $\neg A$ would immediately raise a contradiction. From the remaining disjunctions we can choose the first disjuncts without causing a clash:

$$v_0 : \begin{array}{l} \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B \\ \ \exists r.A, \neg B, \neg C\} \end{array}$$

We apply the $\exists$-Rule on $\exists r.A$ now and add $C_{\mathcal{T}}$ as well as the conjunctions (in $C_{\mathcal{T}}$) to the label of v_1:

$$\begin{tikzcd}
v_0 \arrow[d, dash, "r"] & \begin{array}{l} \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B \\ \ \exists r.A, \neg B, \neg C\} \end{array} \\
v_1 & \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B\}
\end{tikzcd}$$

Now we can propagate B over the r-edge to v_1 by applying the $\forall$-Rule to $\forall r.B$. As well as we can apply the $\sqcup$-Rule (in v_0 and v_1) to all three disjunctions, where we choose the second disjunct $\exists r.A$ from the first, and respectively the first disjunct from the remaining disjunctions. But the choice of $\neg B$ yields a contradiction:

$$\begin{tikzcd}
v_0 \arrow[d, dash, "r"] & \begin{array}{l} \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \ \exists r.A, \neg B, \neg C\} \end{array} \\
v_1 & \begin{array}{l} \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \ B, \exists r.A, \neg B, \neg C\} \end{array}
\end{tikzcd}$$

Thus we revise our previous decision in v_1 and choose $\exists r^-.C$ instead:

$$\begin{tikzcd}
v_0 \arrow[d, dash, "r"] & \begin{array}{l} \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \ \exists r.A, \neg B, \neg C\} \end{array} \\
v_1 & \begin{array}{l} \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \ B, \exists r.A, \exists r^-.C, \neg C\} \end{array}
\end{tikzcd}$$

We apply again the $\exists$-Rule to to $\exists r.A$ and $\exists r^-.C$ in v_1. Again we add $C_{\mathcal{T}}$ to both new nodes, and apply the $\sqcap$-Rule on $C_{\mathcal{T}}$. For the node introduced via $\exists r.A$, we can already add B to its label ($\forall$-Rule), which restricts our choice for the second disjunction:

$$\begin{tikzcd}
v_0 \arrow[d, "r"] & \begin{array}{l} \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \ \exists r.A, \neg B, \neg C\} \end{array} \\
v_1 \arrow[d, "r"] \arrow[dd, "r^-"'] & \begin{array}{l} \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \ B, \exists r.A, \exists r^-.C, \neg C\} \end{array} \\
v_2 & \begin{array}{l} \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \ B, \exists r.A, \exists r^-.C, \neg C\} \end{array} \\
v_3 & \begin{array}{l} \{C, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \ \neg A, \neg B, \forall r.\forall r.B\} \end{array}
\end{tikzcd}$$

v_1 blocks v_2 now (equal labeling), but we can apply the $\forall$-Rule on $\forall r.\forall r.B$ in v_3:

$$\begin{tikzcd}
v_0 \arrow[d, "r"] & \begin{array}{l} \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \ \exists r.A, \neg B, \neg C\} \end{array} \\
v_1 \arrow[d, "r"] \arrow[dd, "r^-"'] & \begin{array}{l} \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \ B, \exists r.A, \exists r^-.C, \neg C, \forall r.B\} \end{array} \\
v_2 & \begin{array}{l} \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \ B, \exists r.A, \exists r^-.C, \neg C\} \end{array} \\
v_3 & \begin{array}{l} \{C, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \ \neg A, \neg B, \forall r.\forall r.B\} \end{array}
\end{tikzcd}$$

where $\forall r.B$ in v_1 is already satisfied, since B is in the label of v_2 which is not longer blocked by v_1. We can therefore apply the $\exists$-Rule on $\exists r^-.C$ and $\exists r.A$ in v_2 (we omit the label of v_3 in the sequel):

$$\begin{tikzcd}
 & v_0 \arrow[d, "r"] & \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \exists r.A, \neg B, \neg C\} \\
 & v_1 \arrow[dl, "r^-"'] \arrow[d, "r"] & \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, B, \exists r.A, \exists r^-.C, \neg C, \forall r.B\} \\
v_3 & v_2 \arrow[d, "r"] \arrow[ddl, "r^-"'] & \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, B, \exists r.A, \exists r^-.C, \neg C\} \\
 & v_4 & \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, B, \exists r.A, \exists r^-.C, \neg C\} \\
v_5 & \{C, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \neg A, \neg B, \forall r.\forall r.B\} &
\end{tikzcd}$$

Again we apply the $\forall$-Rule on $\forall r.\forall r.B$ in v_5, which makes v_2 blocked by v_1 again, as well as v_4 and v_5 are indirectly blocked:

$$\begin{tikzcd}
 & v_0 \arrow[d, "r"] & \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \exists r.A, \neg B, \neg C\} \\
 & v_1 \arrow[dl, "r^-"'] \arrow[d, "r"] & \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, B, \exists r.A, \exists r^-.C, \neg C, \forall r.B\} \\
v_3 & v_2 \arrow[d, "r"] \arrow[ddl, "r^-"'] & \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, B, \exists r.A, \exists r^-.C, \neg C, \forall r.B\} \\
 & v_4 & \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, B, \exists r.A, \exists r^-.C, \neg C\} \\
v_5 & \{C, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \neg A, \neg B, \forall r.\forall r.B\} &
\end{tikzcd}$$

The tableau terminates at this point, since no rule can be applied anymore.

Solution (4.4)**.**
We reconstruct what Markus has done. We have:

$$C_{\mathcal{T}} = (\neg A \sqcup (\exists r^-.A \sqcap \exists r.B)) \sqcap \leqslant 1\, r$$

and initialize a tableau with a single node v_0, labeled with $B \sqcap \exists r^-.A$. We add $C_{\mathcal{T}}$ and apply the $\sqcap$-Rule. We choose $\neg A$ for the $\sqcup$-Rule application and obtain:

$$v_0 \quad \{B \sqcap \exists r^-.A, C_{\mathcal{T}}, B, \exists r^-.A, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \neg A\}$$

Then we apply the $\exists$-Rule on $\exists r^-.A$, apply the $\sqcap$- and $\sqcup$-Rule, but we can not choose $\neg A$ without causing a contradiction:

$$\begin{tikzcd}
v_0 \arrow[d, "r^-"] & \{B \sqcap \exists r^-.A, C_{\mathcal{T}}, B, \exists r^-.A, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \neg A\} \\
v_1 & \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\}
\end{tikzcd}$$

The same steps for $\exists r.A$ we reconsider in v_1 ($\exists r.B$ is already satisfied in v_0):

$$
\begin{array}{ll}
v_0 & \{B \sqcap \exists r^-.A, C_{\mathcal{T}}, B, \exists r^-.A, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \neg A\} \\
\big\downarrow r^- & \\
v_1 & \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} \\
\big\downarrow r^- & \\
v_2 & \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\}
\end{array}
$$

Finally we constructed the situation Markus has ended up in. But what Markus has forgotten, is that we have to use pairwise-blocking when dealing with inverse (r^-) and functional ($\leqslant 1\, r$) roles. Thus, we need to continue and apply the $\exists$-Rule on the two existentials in v_2 ($\exists r^-.B$ is not satisfied by the predecessor anymore):

$$
\begin{tikzcd}[row sep=small]
 & v_0 \arrow[d, "r^-"] & \{B \sqcap \exists r^-.A, C_{\mathcal{T}}, B, \exists r^-.A, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \neg A\} \\
 & v_1 \arrow[d, "r^-"] & \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} \\
 & v_2 \arrow[d, "r^-"] \arrow[ddl, "r^-"'] & \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} \\
 & v_3 & \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} \\
v_4 & \{B, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \neg A\} &
\end{tikzcd}
$$

The restriction $\leqslant 1\, r$ for v_2 is now violated (flip the inverse roles in mind). We therefore have to merge v_4 and v_1, leading to contradiction, such that we first choose the other disjunct in v_4, $(\exists r - \cdot A \sqcap \exists r.B)$:

$$
\begin{tikzcd}[row sep=small]
 & v_0 \arrow[d, "r^-"] & \{B \sqcap \exists r^-.A, C_{\mathcal{T}}, B, \exists r^-.A, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \neg A\} \\
 & v_1 \arrow[d, "r^-"] & \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} \\
 & v_2 \arrow[d, "r^-"] \arrow[ddl, "r^-"'] & \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} \\
 & v_3 & \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} \\
v_4 & \{B, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} &
\end{tikzcd}
$$

Merging v_4 with v_1 is now possible, for which we add B to v_1:

$$
\begin{array}{ll}
v_0 & \{B \sqcap \exists r^-.A, C_{\mathcal{T}}, B, \exists r^-.A, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \neg A\} \\
\big\downarrow r^- & \\
v_1 & \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B, B\} \\
\big\downarrow r^- & \\
v_2 & \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} \\
\big\downarrow r^- & \\
v_3 & \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\}
\end{array}
$$

We still can not reach a blocking situation and therefore generate successors for v_3 ($\rightsquigarrow$ applying the $\exists$-Rule):

$$\begin{tikzcd}[row sep=small]
 & v_0 \arrow[d, "r^-"] & \{B \sqcap \exists r^-.A, C_\mathcal{T}, B, \exists r^-.A, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \neg A\} \\
 & v_1 \arrow[d, "r^-"] & \{A, C_\mathcal{T}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B, B\} \\
 & v_2 \arrow[d, "r^-"] & \{A, C_\mathcal{T}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} \\
 & v_3 \arrow[d, "r^-"] \arrow[ddl, "r"'] & \{A, C_\mathcal{T}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} \\
 & v_4 & \{A, C_\mathcal{T}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} \\
v_5 & \{B, C_\mathcal{T}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} &
\end{tikzcd}$$

Since $\leqslant 1\, r$ is violated again in v_3 this time, we need to merge v_5 with v_2. We make the same choice for the disjunction in v_5 and merge:

$$\begin{array}{ll}
v_0 & \{B \sqcap \exists r^-.A, C_\mathcal{T}, B, \exists r^-.A, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \neg A\} \\
\downarrow r^- & \\
v_1 & \{A, C_\mathcal{T}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B, B\} \\
\downarrow r^- & \\
v_2 & \{A, C_\mathcal{T}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B, B\} \\
\downarrow r^- & \\
v_3 & \{A, C_\mathcal{T}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} \\
\downarrow r^- & \\
v_4 & \{A, C_\mathcal{T}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\}
\end{array}$$

We conduct the previous two steps again (introduce successors of v_4 and then merge) and obtain:

$$\begin{array}{ll}
v_0 & \{B \sqcap \exists r^-.A, C_\mathcal{T}, B, \exists r^-.A, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \neg A\} \\
\downarrow r^- & \\
v_1 & \{A, C_\mathcal{T}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B, B\} \\
\downarrow r^- & \\
v_2 & \{A, C_\mathcal{T}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B, B\} \\
\downarrow r^- & \\
v_3 & \{A, C_\mathcal{T}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B, B\} \\
\downarrow r^- & \\
v_4 & \{A, C_\mathcal{T}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\} \\
\downarrow r^- & \\
v_5 & \{A, C_\mathcal{T}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leqslant 1\, r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\}
\end{array}$$

Ultimately we can derive a blocking situation with the pairs (v_1, v_2) and (v_2, v_3), hence $x = v_2$ and predecessor $x' = v_1$ blocks $y = v_3$ with predecessor $y' = v_2$, and therefore v_2 blocks v_3 directly.

Markus mistake was to not consider (user) pairwise blocking when dealing with inverse roles in combination with functional ones.

Solution (4.5)**.**
Axioms of the form $\top \sqsubseteq \leqslant 1\, r.A$ are transformed in $C_\mathcal{T}$ (NNF) to $\sqsubseteq \leqslant 1\, r.A$, where r is referred to be functional in the $\leqslant 1$-Rule. We can extend the rule as follows:

$\leqslant 1$-Rule: For some $v \in V$ with $\leqslant 1\, r.A \in L(v)$ and two f-neighbors v_1 and v_2 with $A \in L(v_1) \cap L(v_2)$, merge(v_1, v_2).