

Foundations of Semantic Web Technologies

Solutions for Tutorial 4: Tableau

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Solution (4.1).

$$(a) \text{ NNF}(\neg(A \sqcap \forall r.B)) \Rightarrow \neg A \sqcup \exists r.(\neg B)$$

$$(b) \text{ NNF}(\neg\forall r.\exists s.(\neg B \sqcup \exists r.A)) \Rightarrow \exists r.\forall s.(B \sqcap \forall r.(\neg A))$$

$$(c) \text{ NNF}(\neg((\neg A \sqcap \exists r.\top) \sqcup \geq 3 s.(A \sqcup \neg B))) \Rightarrow (\neg(\neg A \sqcap \exists r.\top) \sqcap \neg(\geq 3 s.(A \sqcup \neg B))) \Rightarrow ((A \sqcup \forall r.\perp) \sqcap \leq 2 s.(A \sqcup \neg B))$$

Solution (4.2).

We want to test, whether A is a subset of B in all models of the knowledge base. Since we can not construct all models, we reformulate the (subsumption) problem and test now for the complement $A \sqcap \neg B$, thereby requesting a model where an individual is in A but not in B . If such a model can be determined, the subsumption does NOT hold, otherwise we conclude that $A \sqsubseteq B$ is entailed by the knowledge base.

$C_{\mathcal{T}} = (C \sqcup B) \sqcap (\neg A \sqcup \neg C \sqcup \perp)$ simplified $C_{\mathcal{T}} = (C \sqcup B) \sqcap (\neg A \sqcup \neg C)$. The tableau is initialized with node v_0 , labeled with A and $\neg B$.

$$v_0 : \{A, \neg B, C_{\mathcal{T}}, C \sqcup B, \neg A \sqcup \neg C\}$$

We try it with the first disjunct of each disjunction:

$$v_0 : \{A, \neg B, C_{\mathcal{T}}, C \sqcup B, \neg A \sqcup \neg C, C, \neg A\}$$

Which yields the contradiction A and $\neg A$, and we try $\neg C$ instead of $\neg A$:

$$v_0 : \{A, \neg B, C_{\mathcal{T}}, C \sqcup B, \neg A \sqcup \neg C, C, \neg C\}$$

But again, this yields the contradiction C and $\neg C$, which forces us to consider the first disjunction again and choose B .

$$v_0 : \{A, \neg B, C_{\mathcal{T}}, C \sqcup B, \neg A \sqcup B\}$$

This also yields a contradiction and we can not apply any rule further, which leaves us with a closed tableau. Consequently, the subsumption $A \sqsubseteq B$ holds, i.e. it is entailed by the TBox.

Solution (4.3).

Since we deal with existential quantifiers now, we need to consider blocking. And since also inverse roles are used, we have to use equality blocking.

$$C_{\mathcal{T}} = (\neg A \sqcup \exists r.A) \sqcap (\neg B \sqcup \exists r.^{\neg}.C) \sqcap (\neg C \sqcup \forall r.\forall r.B)$$

The tableau is initialized with node v_0 labeled by $\{A \sqcap \forall r.B\}$ and $C_{\mathcal{T}}$. Applying the \sqcap -Rule yields:

$$v_0 : \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r.^{\neg}.C, \neg C \sqcup \forall r.\forall r.B\}$$

Considering the disjunctions, we choose $\exists r.A$ from the first one since $\neg A$ would immediately raise a contradiction. From the remaining disjunctions we can choose the first disjuncts without causing a clash:

$$v_0 : \begin{array}{l} \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B \\ \exists r.A, \neg B, \neg C\} \end{array}$$

We apply the \exists -Rule on $\exists r.A$ now and add $C_{\mathcal{T}}$ as well as the conjunctions (in $C_{\mathcal{T}}$) to the label of v_1 :

$$\begin{array}{l} v_0 \quad \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B \\ \quad \exists r.A, \neg B, \neg C\} \\ \quad \downarrow r \\ v_1 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B\} \end{array}$$

Now we can propagate B over the r -edge to v_1 by applying the \forall -Rule to $\forall r.B$. As well as we can apply the \sqcup -Rule (in v_0 and v_1) to all three disjunctions, where we choose the second disjunct $\exists r.A$ from the first, and respectively the first disjunct from the remaining disjunctions. But the choice of $\neg B$ yields a contradiction:

$$\begin{array}{l} v_0 \quad \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \quad \exists r.A, \neg B, \neg C\} \\ \quad \downarrow r \\ v_1 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \quad B, \exists r.A, \neg B, \neg C\} \end{array}$$

Thus we revise our previous decision in v_1 and choose $\exists r^-.C$ instead:

$$\begin{array}{l} v_0 \quad \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \quad \exists r.A, \neg B, \neg C\} \\ \quad \downarrow r \\ v_1 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \quad B, \exists r.A, \exists r^-.C, \neg C\} \end{array}$$

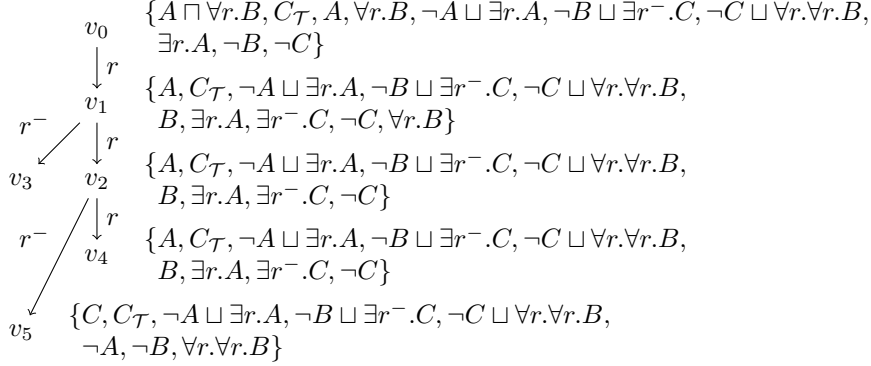
We apply again the \exists -Rule to $\exists r.A$ and $\exists r^-.C$ in v_1 . Again we add $C_{\mathcal{T}}$ to both new nodes, and apply the \sqcap -Rule on $C_{\mathcal{T}}$. For the node introduced via $\exists r.A$, we can already add B to its label (\forall -Rule), which restricts our choice for the second disjunction:

$$\begin{array}{l} v_0 \quad \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \quad \exists r.A, \neg B, \neg C\} \\ \quad \downarrow r \\ v_1 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \quad B, \exists r.A, \exists r^-.C, \neg C\} \\ \quad \downarrow r \\ v_2 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \quad B, \exists r.A, \exists r^-.C, \neg C\} \\ \quad \downarrow r^- \\ v_3 \quad \{C, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \quad \neg A, \neg B, \forall r.\forall r.B\} \end{array}$$

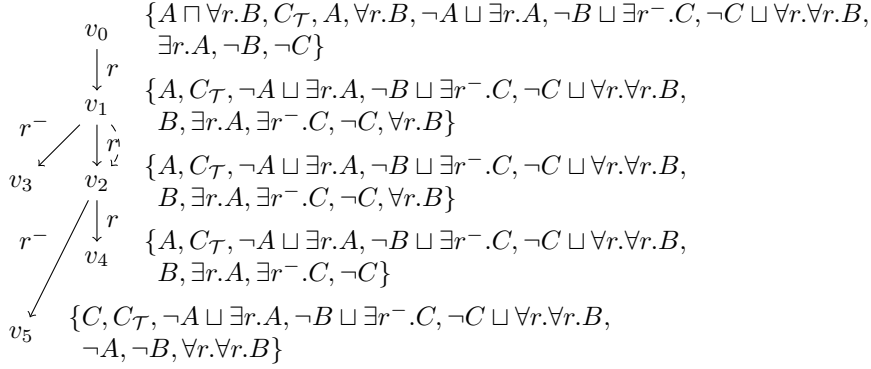
v_1 blocks v_2 now (equal labeling), but we can apply the \forall -Rule on $\forall r.\forall r.B$ in v_3 :

$$\begin{array}{l} v_0 \quad \{A \sqcap \forall r.B, C_{\mathcal{T}}, A, \forall r.B, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \quad \exists r.A, \neg B, \neg C\} \\ \quad \downarrow r \\ v_1 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \quad B, \exists r.A, \exists r^-.C, \neg C, \forall r.B\} \\ \quad \downarrow r \\ v_2 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \quad B, \exists r.A, \exists r^-.C, \neg C\} \\ \quad \downarrow r^- \\ v_3 \quad \{C, C_{\mathcal{T}}, \neg A \sqcup \exists r.A, \neg B \sqcup \exists r^-.C, \neg C \sqcup \forall r.\forall r.B, \\ \quad \neg A, \neg B, \forall r.\forall r.B\} \end{array}$$

where $\forall r.B$ in v_1 is already satisfied, since B is in the label of v_2 which is not longer blocked by v_1 . We can therefore apply the \exists -Rule on $\exists r^-.C$ and $\exists r.A$ in v_2 (we omit the label of v_3 in the sequel):



Again we apply the \forall -Rule on $\forall r.\forall r.B$ in v_5 , which makes v_2 blocked by v_1 again, as well as v_4 and v_5 are indirectly blocked:



The tableau terminates at this point, since no rule can be applied anymore.

Solution (4.4).

We reconstruct what Markus has done. We have:

$$C_{\mathcal{T}} = (\neg A \sqcup (\exists r^-.A \sqcap \exists r.B)) \sqcap \leq 1 r$$

and initialize a tableau with a single node v_0 , labeled with $B \sqcap \exists r^-.A$. We add $C_{\mathcal{T}}$ and apply the \sqcap -Rule. We choose $\neg A$ for the \sqcup -Rule application and obtain:

$$v_0 \quad \{B \sqcap \exists r^-.A, C_{\mathcal{T}}, B, \exists r^-.A, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leq 1 r, \neg A\}$$

Then we apply the \exists -Rule on $\exists r^-.A$, apply the \sqcap - and \sqcup -Rule, but we can not choose $\neg A$ without causing a contradiction:

$$\begin{array}{l}
v_0 \quad \{B \sqcap \exists r^-.A, C_{\mathcal{T}}, B, \exists r^-.A, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leq 1 r, \neg A\} \\
\downarrow r^- \\
v_1 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^-.A \sqcap \exists r.B), \leq 1 r, \exists r^-.A \sqcap \exists r.B, \exists r^-.A, \exists r.B\}
\end{array}$$

The same steps for $\exists r.A$ we reconsider in v_1 ($\exists r.B$ is already satisfied in v_0):

$$\begin{array}{l}
v_0 \quad \{B \sqcap \exists r^- . A, C_{\mathcal{T}}, B, \exists r^- . A, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \neg A\} \\
\downarrow r^- \\
v_1 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \exists r^- . A \sqcap \exists r . B, \exists r^- . A, \exists r . B\} \\
\downarrow r^- \\
v_2 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \exists r^- . A \sqcap \exists r . B, \exists r^- . A, \exists r . B\}
\end{array}$$

Finally we constructed the situation Markus has ended up in. But what Markus has forgotten, is that we have to use pairwise-blocking when dealing with inverse (r^-) and functional ($\leq 1 r$) roles. Thus, we need to continue and apply the \exists -Rule on the two existentials in v_2 ($\exists r^- . B$ is not satisfied by the predecessor anymore):

$$\begin{array}{l}
v_0 \quad \{B \sqcap \exists r^- . A, C_{\mathcal{T}}, B, \exists r^- . A, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \neg A\} \\
\downarrow r^- \\
v_1 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \exists r^- . A \sqcap \exists r . B, \exists r^- . A, \exists r . B\} \\
\downarrow r^- \\
v_2 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \exists r^- . A \sqcap \exists r . B, \exists r^- . A, \exists r . B\} \\
\begin{array}{l} \swarrow r^- \\ \downarrow r^- \\ \searrow r^- \end{array} \\
v_3 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \exists r^- . A \sqcap \exists r . B, \exists r^- . A, \exists r . B\} \\
\swarrow r^- \\
v_4 \quad \{B, C_{\mathcal{T}}, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \neg A\}
\end{array}$$

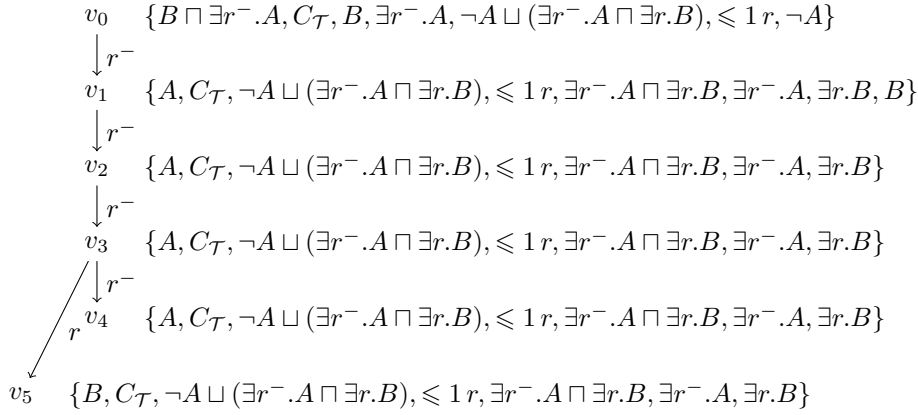
The restriction $\leq 1 r$ for v_2 is now violated (flip the inverse roles in mind). We therefore have to merge v_4 and v_1 , leading to contradiction, such that we first choose the other disjunct in v_4 , ($\exists r^- . A \sqcap \exists r . B$):

$$\begin{array}{l}
v_0 \quad \{B \sqcap \exists r^- . A, C_{\mathcal{T}}, B, \exists r^- . A, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \neg A\} \\
\downarrow r^- \\
v_1 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \exists r^- . A \sqcap \exists r . B, \exists r^- . A, \exists r . B\} \\
\downarrow r^- \\
v_2 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \exists r^- . A \sqcap \exists r . B, \exists r^- . A, \exists r . B\} \\
\begin{array}{l} \swarrow r^- \\ \downarrow r^- \\ \searrow r^- \end{array} \\
v_3 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \exists r^- . A \sqcap \exists r . B, \exists r^- . A, \exists r . B\} \\
\swarrow r^- \\
v_4 \quad \{B, C_{\mathcal{T}}, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \exists r^- . A \sqcap \exists r . B, \exists r^- . A, \exists r . B\}
\end{array}$$

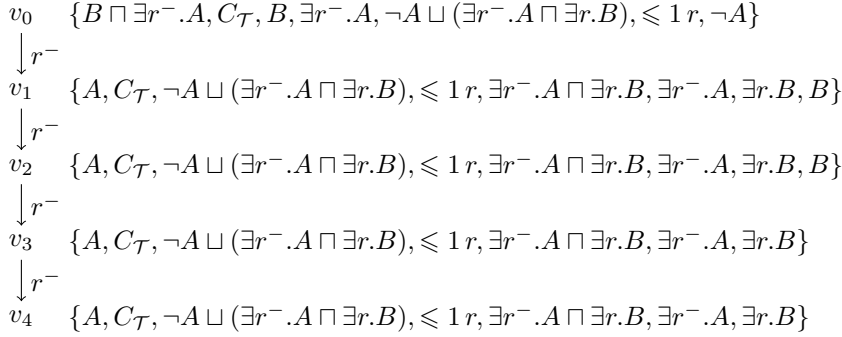
Merging v_4 with v_1 is now possible, for which we add B to v_1 :

$$\begin{array}{l}
v_0 \quad \{B \sqcap \exists r^- . A, C_{\mathcal{T}}, B, \exists r^- . A, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \neg A\} \\
\downarrow r^- \\
v_1 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \exists r^- . A \sqcap \exists r . B, \exists r^- . A, \exists r . B, B\} \\
\downarrow r^- \\
v_2 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \exists r^- . A \sqcap \exists r . B, \exists r^- . A, \exists r . B\} \\
\downarrow r^- \\
v_3 \quad \{A, C_{\mathcal{T}}, \neg A \sqcup (\exists r^- . A \sqcap \exists r . B), \leq 1 r, \exists r^- . A \sqcap \exists r . B, \exists r^- . A, \exists r . B\}
\end{array}$$

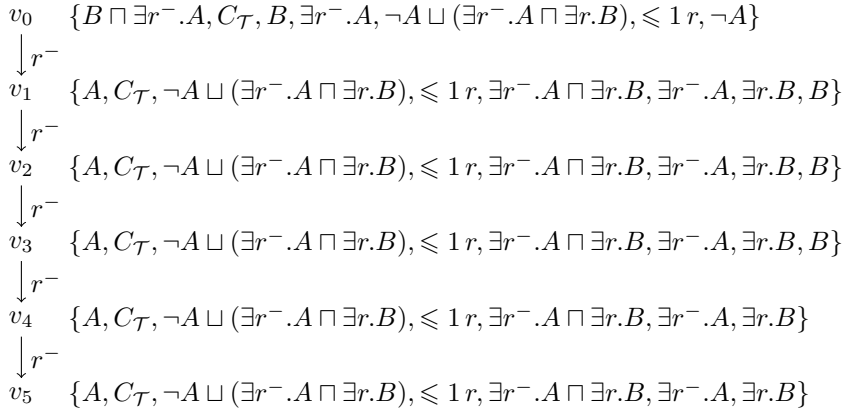
We still can not reach a blocking situation and therefore generate successors for v_3 (\rightsquigarrow applying the \exists -Rule):



Since $\leq 1 r$ is violated again in v_3 this time, we need to merge v_5 with v_2 . We make the same choice for the disjunction in v_5 and merge:



We conduct the previous two steps again (introduce successors of v_4 and then merge) and obtain:



Ultimately we can derive a blocking situation with the pairs (v_1, v_2) and (v_2, v_3) , hence $x = v_2$ and predecessor $x' = v_1$ blocks $y = v_3$ with predecessor $y' = v_2$, and therefore v_2 blocks v_3 directly.

Markus mistake was to not consider (user) pairwise blocking when dealing with inverse roles in combination with functional ones.

Solution (4.5).

Axioms of the form $\top \sqsubseteq \leq 1 r.A$ are transformed in $C_{\mathcal{T}}$ (NNF) to $\sqsubseteq \leq 1 r.A$, where r is referred to be functional in the ≤ 1 -Rule. We can extend the rule as follows:

≤ 1 -Rule: For some $v \in V$ with $\leq 1 r.A \in L(v)$ and two f -neighbors v_1 and v_2 with $A \in L(v_1) \cap L(v_2)$, $\text{merge}(v_1, v_2)$.