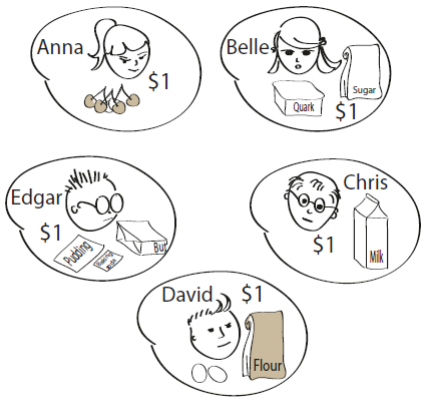
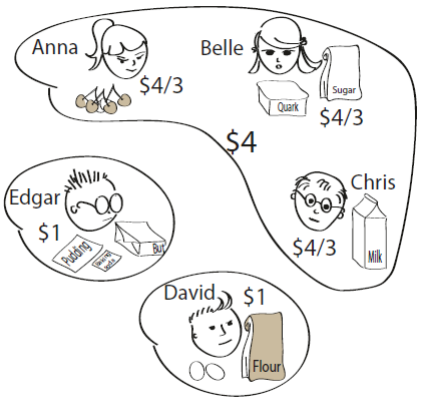
Algorithmic Game Theory
Introduction to Strategic Games - Problems 10

10.07.2023

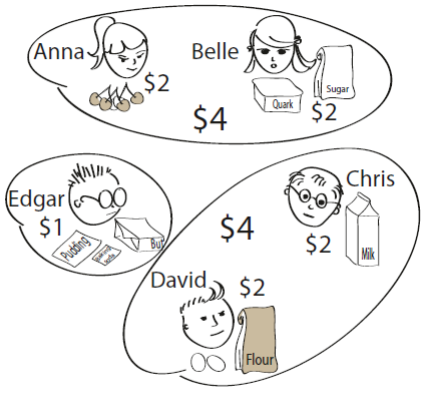
Problem 1. *Consider the following four coalition structures:*



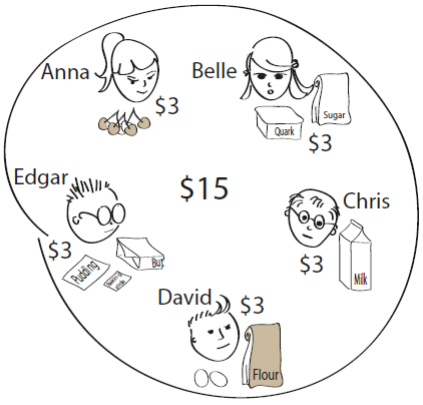
(a) Coalition structure $\mathfrak{C}_1$



(b) Coalition structure $\mathfrak{C}_2$



(c) Coalition structure $\mathfrak{C}_3$



(d) Coalition structure $\mathfrak{C}_4$

Write down the four coalition structures formally. Then, write down the amount each player receives for each outcome. Finally, argue from the coalitions displayed here whether the characteristic function is monotonic.

Problem 2. *Three players together can obtain* \$1 *to share, any two players can obtain* α*, and one player by herself can obtain zero.*

Then $v(N) = 1, v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = \alpha, v(\{1\}) = v(\{2\}) = v(\{3\}) = v(\emptyset) = 0$

What is the core of this game? When is the core empty? When is the core non-empty?

Problem 3. *Consider the 3-person game in which no single player can generate any surplus on her own:*

$$\begin{aligned} &v(\{1\}) = 0, v(\{2\}) = 0, v(\{3\}) = 0, \\ &v(\{1,2\}) = 7, v(\{1,3\}) = 6, v(\{2,3\}) = 5, \\ &v(N) = 8 \end{aligned}$$

Is the core non-empty? If not, what is the cost of stability/the value of the least core?

Problem 4. *Consider the 3-person game with* $v(C) = 2$ *if* $|C| \geq 2$ *and v(C) = 0 otherwise. Show that* $\{(1,1,0),(1,0,1),(0,1,1)\}$ *is a stable set of this game.*

Problem 5. *(Bonus Problem) Show that the game from Problem 4 has (at least) one other stable set.*

Hint: Consider the stable set given for "Hospitals and X-Ray Machines" in the lecture.