## Complexity Theory NP-Complete Problems

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**Computational Logic** 

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#### **Review**

#### Further NP-complete Problems

#### **Towards More NP-Complete Problems**

Starting with SAT, one can readily show more problems  $\mathcal{P}$  to be NP-complete, each time performing two steps:

- (1) Show that  $\mathcal{P} \in NP$
- (2) Find a known NP-complete problem  $\mathcal{P}'$  and reduce  $\mathcal{P}' \leq_p \mathcal{P}$

Thousands of problem have now been shown to be NP-complete. (See Garey and Johnson for an early survey)

In this course:

 $\leq_{p} \text{Clique} \leq_{p} \text{Independent Set}$ Sat  $\leq_{p} 3$ -Sat  $\leq_{p} \text{Dir. Hamiltonian Path}$  $\leq_{p} \text{Subset Sum} \leq_{p} \text{Knapsack}$ 

#### DIRECTED HAMILTONIAN PATH

*Input:* A directed graph G.

*Problem:* Is there a directed path in *G* containing every vertex exactly once?

Theorem 9.1

DIRECTED HAMILTONIAN PATH *is* NP*-complete*.

DIRECTED	HAMILTONIAN	Ратн
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#### Proof.

• DIRECTED HAMILTONIAN PATH  $\in NP$ : Take the path to be the certificate.

#### Digression: How to design reductions

Task: Show that problem  $\mathcal{P}$  (DIR. HAMILTONIAN PATH) is NP-hard.

Arguably, the most important part is to decide where to start from.

That is, which problem to reduce to DIRECTED HAMILTONIAN PATH?

- Considerations:
  - ► Is there an NP-complete problem similar to *P*? (for example, CLIQUE and INDEPENDENT SET)
  - It is not always beneficial to choose a problem of the same type

(for example, reducing a graph problem to a graph problem)

- For instance, CLIQUE, INDEPENDENT SET are "local" problems (is there a set of vertices inducing some structure)
- Hamiltonian Path is a global problem (find a structure – the Hamiltonian path – containing all vertices)

### Digression: How to design reductions

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  - ► Is there an NP-complete problem similar to *P*? (for example, CLIQUE and INDEPENDENT SET)
  - It is not always beneficial to choose a problem of the same type

(for example, reducing a graph problem to a graph problem)

- ► For instance, CLIQUE, INDEPENDENT SET are "local" problems (is there a set of vertices inducing some structure)
- Hamiltonian Path is a global problem (find a structure – the Hamiltonian path – containing all vertices)
- How to design the reduction:
  - Does your problem come from an optimisation problem? If so: a maximisation problem? a minimisation problem?
  - Learn from examples, have good ideas.

DIRECTED	HAMILTONIAN	Ратн
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- ► DIRECTED HAMILTONIAN PATH IS NP-hard: 3-Sat ≤<sub>D</sub> Directed Hamiltonian Path

Proof idea: (see blackboard for details) Let  $\varphi := \bigwedge_{i=1}^{k} C_i$  and  $C_i := (L_{i,1} \lor L_{i,2} \lor L_{i,3})$ 

- For each variable X occurring in φ, we construct a directed graph ("gadget") that allows only two Hamiltonian paths: "true" and "false"
- Gadgets for each variable are "chained" in a directed fashion, so that all variables must be assigned one value
- Clauses are represented by vertices that are connected to the gadgets in such a way that they can only be visited on a Hamiltonian path that corresponds to an assignment where they are true

Details are also given in [Sipser, Theorem 7.46].

#### Example 9.2 (see blackboard)

 $\varphi := C_1 \land C_2$  where  $C_1 := (X \lor \neg Y \lor Z)$  and  $C_2 := (\neg X \lor Y \lor \neg Z)$ 

#### **Towards More NP-Complete Problems**

Starting with SAT, one can readily show more problems  $\mathcal{P}$  to be NP-complete, each time performing two steps:

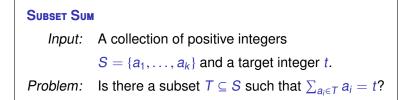
- (1) Show that  $\mathcal{P} \in NP$
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## NP-Completeness of SUBSET SUM



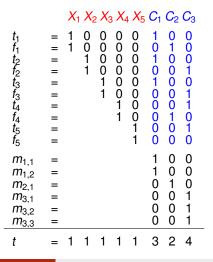
Theorem 9.3

SUBSET SUM *is* NP*-complete*.

Proof.

- SUBSET SUM  $\in$  NP: Take T to be the certificate.
- SUBSET SUM is NP-hard: SAT  $\leq_p$  SUBSET SUM

#### $(X_1 \lor X_2 \lor X_3) \land (\neg X_1 \lor \neg X_4) \land (X_4 \lor X_5 \lor \neg X_2 \lor \neg X_3)$



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## Sat $\leq_p$ Subset Sum

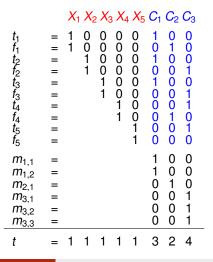
**Given:**  $\varphi := C_1 \land \cdots \land C_k$  in conjunctive normal form.

(w.l.o.g. at most 9 literals per clause)

Let  $X_1, \ldots, X_n$  be the variables in  $\varphi$ . For each  $X_i$  let

$$t_i := a_1 \dots a_n c_1 \dots c_k \text{ where } a_j := \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \text{ and } c_j := \begin{cases} 1 & X_i \text{ occurs in } C_j \\ 0 & \text{otherwise} \end{cases}$$
$$f_i := a_1 \dots a_n c_1 \dots c_k \text{ where } a_j := \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \text{ and } c_j := \begin{cases} 1 & \neg X_i \text{ occurs in } C_j \\ 0 & \text{otherwise} \end{cases}$$

#### $(X_1 \lor X_2 \lor X_3) \land (\neg X_1 \lor \neg X_4) \land (X_4 \lor X_5 \lor \neg X_2 \lor \neg X_3)$



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### Sat $\leq_p$ Subset Sum

Further, for each clause  $C_i$  take  $r := |C_i| - 1$  integers  $m_{i,1}, \ldots, m_{i,r}$ 

where 
$$m_{i,j} := c_i \dots c_k$$
 with  $c_\ell := \begin{cases} 1 & \ell = i \\ 0 & \ell \neq i \end{cases}$   
Definition of *S*: Let

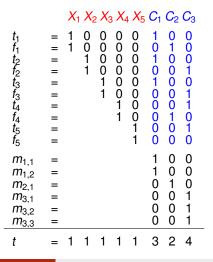
 $S := \{t_i, f_i \mid 1 \le i \le n\} \cup \{m_{i,j} \mid 1 \le i \le k, \quad 1 \le j \le |C_i| - 1\}$ 

Target: Finally, choose as target

 $t := a_1 \dots a_n c_1 \dots c_k$  where  $a_i := 1$  and  $c_i := |C_i|$ 

Claim: There is  $T \subseteq S$  with  $\sum_{a_i \in T} a_i = t$  iff  $\varphi$  is satisfiable.

#### $(X_1 \lor X_2 \lor X_3) \land (\neg X_1 \lor \neg X_4) \land (X_4 \lor X_5 \lor \neg X_2 \lor \neg X_3)$



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## NP-Completeness of SUBSETSUM

Let  $\varphi := \bigwedge C_i$   $C_i$ : clauses

Show: If  $\varphi$  is satisfiable, then there is  $T \subseteq S$  with  $\sum_{s \in T} s = t$ .

Let  $\beta$  be a satisfying assignment for  $\varphi$ 

Set 
$$T_1 := \{t_i \mid \beta(X_i) = 1 \quad 1 \le i \le m\} \cup \{f_i \mid \beta(X_i) = 0 \quad 1 \le i \le m\}$$

Further, for each clause  $C_i$  let  $r_i$  be the number of satisfied literals in  $C_i$  (with resp. to  $\beta$ ).

Set 
$$T_2 := \{m_{i,j} \mid 1 \le i \le k, 1 \le j \le |C_i| - r_i\}$$

and define  $T := T_1 \cup T_2$ .

It follows:  $\sum_{s \in T} s = t$ 

#### NP-Completeness of SUBSET SUM

Show: If there is  $T \subseteq S$  with  $\sum_{s \in T} s = t$ , then  $\varphi$  is satisfiable.

Let  $T \subseteq S$  such that  $\sum_{s \in T} s = t$ 

Define 
$$\beta(X_i) = \begin{cases} 1 & \text{if } t_i \in T \\ 0 & \text{if } f_i \in T \end{cases}$$

This is well defined as for all *i*:  $t_i \in T$  or  $f_i \in T$  but not both.

Further, for each clause, there must be one literal set to 1 as for all *i*, the  $m_{i,j} \in S$  do not sum up to the number of literals in the clause.

#### **Knapsack and Strong NP-Completeness**

#### **Towards More NP-Complete Problems**

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## NP-completeness of KNAPSACK

Knapsack	
Input:	A set $I := \{1, \ldots, n\}$ of items
	each of value $v_i$ and weight $w_i$ for $1 \le i \le n$ ,
	target value $t$ and weight limit $\ell$
Problem:	Is there $T \subseteq I$ such that
	$\sum_{i \in T} v_i \ge t$ and $\sum_{i \in T} w_i \le \ell$ ?

Theorem 9.4

Кларsаск is NP-complete.

#### NP-completeness of KNAPSACK

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Theorem 9.4

Кларsаск is NP-complete.

Proof.

- KNAPSACK  $\in$  NP: Take T to be the certificate.
- ► KNAPSACK IS NP-hard: Subset Sum ≤<sub>p</sub> Knapsack

## Subset Sum $\leq_p$ Knapsack

## Subset Sum:Given: $S := \{a_1, \dots, a_n\}$ collection of positive integersttarget integer

Problem: Is there a subset  $T \subseteq S$  such that  $\sum_{a_i \in T} a_i = t$ ?

## Subset Sum $\leq_p$ Knapsack

# Subset Sum:Given: $S := \{a_1, \dots, a_n\}$ collection of positive integersttarget integer

Problem: Is there a subset  $T \subseteq S$  such that  $\sum_{a_i \in T} a_i = t$ ?

Reduction: From this input to SUBSET SUM construct

- set of items *l* := {1,..., *n*}
- weights and values  $v_i = w_i = a_i$  for all  $1 \le i \le n$
- target value t' := t and weight limit  $\ell := t$

## Subset Sum $\leq_p$ Knapsack

#### Subset Sum:

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Clearly: For every  $T \subseteq S$ 

$$\sum_{a_i \in T} a_i = t \quad \text{iff} \quad \sum_{a_i \in T} v_i \ge t' = t$$
$$\sum_{a_i \in T} w_i \le \ell = t$$

Hence: The reduction is correct and in polynomial time.

Complexity Theory

## A Polynomial Time Algorithm for KNAPSACK

Кларваск can be solved in time  $O(n\ell)$  using dynamic programming

Initialisation:

- Create an  $(\ell + 1) \times (n + 1)$  matrix M
- Set M(w, 0) := 0 for all  $1 \le w \le \ell$  and M(0, i) := 0 for all  $1 \le i \le n$

Input  $I = \{1, 2, 3, 4\}$  with Values:  $v_1 = 1$   $v_2 = 3$   $v_3 = 4$   $v_4 = 2$ 

Weight:  $w_1 = 1$   $w_2 = 1$   $w_3 = 3$   $w_4 = 2$ 

Weight limit:  $\ell = 5$  Target value: t = 7

weight	max. total value from first <i>i</i> items				
limit w	<i>i</i> = 0	<i>i</i> = 1	<i>i</i> = 2	<i>i</i> = 3	<i>i</i> = 4
0					
1					
2					
3					
4					
5					

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0	0	0	0	0	0	
1	0					
2	0					
3	0					
4	0					
5	0					

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Computation: Assign further M(w, i) to be the largest total value obtainable by selecting from the first *i* items with weight limit *w*: For i = 0, 1, ..., n - 1 set M(w, i + 1) as

 $M(w, i+1) := \max \{ M(w, i), M(w - w_{i+1}, i) + v_{i+1} \}$ 

Here, if  $w - w_{i+1} < 0$  we always take M(w, i).

Acceptance: If *M* contains an entry  $\geq t$ , accept. Otherwise reject.

Input  $I = \{1, 2, 3, 4\}$  with Values:  $v_1 = 1$   $v_2 = 3$   $v_3 = 4$   $v_4 = 2$ Weight:  $w_1 = 1$   $w_2 = 1$   $w_3 = 3$   $w_4 = 2$ 

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0	0	0	0	0	0
1	0				
2	0				
3	0				
4	0				
5	0				

Input  $I = \{1, 2, 3, 4\}$  with Values:  $v_1 = 1$   $v_2 = 3$   $v_3 = 4$   $v_4 = 2$ Weight:  $w_1 = 1$   $w_2 = 1$   $w_3 = 3$   $w_4 = 2$ 

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1	0	1			
2	0	1			
3	0	1			
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0	0	0	0	0	0
1	0	1	3		
2	0	1			
3	0	1			
4	0	1			
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1	0	1	3		
2	0	1	4		
3	0	1			
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0	0	0	0	0	0
1	0	1	3		
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0	0	0	0	0	0
1	0	1	3		
2	0	1	4		
3	0	1	4		
4	0	1	4		
5	0	1			

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0	0	0	0	0	0
1	0	1	3		
2	0	1	4		
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0	0	0	0	0	0
1	0	1	3	3	3
2	0	1	4	4	4
3	0	1	4	4	5
4	0	1	4	7	7
5	0	1	4	8	8

#### Did we prove P = NP?

Summary:

- ► Theorem 9.4: Кларѕаск is NP-complete
- KNAPSACK can be solved in time  $O(n\ell)$  using dynamic programming

What went wrong?

Knapsack	
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#### **Pseudo-Polynomial Time**

The previous algorithm is not sufficient to show that KNAPSACK is in  ${\rm P}$ 

- The algorithm fills a  $(\ell + 1) \times (n + 1)$  matrix M
- The size of the input to KNAPSACK is  $O(n \log \ell)$

 $\rightarrow$  the size of *M* is not bounded by a polynomial in the length of the input!

## **Pseudo-Polynomial Time**

The previous algorithm is not sufficient to show that  $\mathsf{K}\mathtt{N}\mathtt{A}\mathtt{P}\mathtt{S}\mathtt{A}\mathtt{C}\mathtt{K}$  is in P

- The algorithm fills a  $(\ell + 1) \times (n + 1)$  matrix M
- The size of the input to KNAPSACK is  $O(n \log \ell)$

 $\rightarrow$  the size of *M* is not bounded by a polynomial in the length of the input!

#### Definition 9.5 (Pseudo-Polynomial Time)

Problems decidable in time polynomial in the sum of the input length and the value of numbers occurring in the input.

Equivalently: Problems decidable in polynomial time when using unary encoding for all numbers in the input.

- If KNAPSACK is restricted to instances with ℓ ≤ p(n) for a polynomial p, then we obtain a problem in P.
- KNAPSACK is in polynomial time for unary encoding of numbers.

## Strong NP-completeness

Pseudo Polynomial time: Algorithms polynomial in the maximum of the input length and the value of numbers occurring in the input.

#### Examples:

- KNAPSACK
- SUBSET SUM

Strong NP-completeness: Problems which remain NP-complete even if all numbers are bounded by a polynomial in the input length (equivalently: even for unary coding of numbers).

Examples:

- CLIQUE
- SAT
- HAMILTONIAN CYCLE

▶ ...

Note: Showing SAT  $\leq_{p}$  SUBSET SUM required exponentially large numbers.

#### CONP

## The Class CONP

Recall that  $\operatorname{CONP}$  is the complement class of NP.

**Definition 9.6** 

- For a language  $\mathcal{L} \subseteq \Sigma^*$  let  $\overline{\mathcal{L}} := \Sigma^* \setminus \mathcal{L}$  be its complement
- For a complexity class C, we define  $\operatorname{coC} := \{\mathcal{L} \mid \overline{\mathcal{L}} \in C\}$
- In particular  $\operatorname{coNP} = \{\mathcal{L} \mid \overline{\mathcal{L}} \in \operatorname{NP}\}$

A problem belongs to  $\operatorname{coNP}$ , if no-instances have short certificates.

#### Examples:

- No HAMILTONIAN PATH: Does the graph G not have a Hamiltonian path?
- TAUTOLOGY: Is the propositional logic formula φ a tautology (true under all assignments)?

#### CONP

## coNP-completeness

**Definition 9.7** 

A language  $C \in \text{coNP}$  is coNP-complete, if  $\mathcal{L} \leq_p C$  for all  $\mathcal{L} \in \text{coNP}$ .

Theorem 9.8

- P = COP
- Hence,  $P \subseteq NP \cap CONP$

Open questions:

 $\triangleright$  NP = coNP?

Most people do not think so.

 $P = NP \cap CONP?$ 

Again, most people do not think so.