Exercise 11: Graph Databases and Path Queries

Database Theory

2020-07-06

Maximilian Marx, David Carral

Exercise 1

Exercise. It was explained in the lecture that RDF and Property Graph can encode the same graph structures. How could we encode arbitrary hypergraphs (relational databases) in RDF? RDF can be considered as a synonym for “labelled directed graph” here – the technical details of the RDF standard are not important for this exercise.

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- we add labels $p_1, p_2, \dots, p_\ell$;
- a vertex v_φ; and
- edges $p_1(c_\varphi, t_1), p_2(c_\varphi, t_2), \dots, p_\ell(c_\varphi, t_\ell)$ to G_{RDF}.

Exercise 2.

Exercise. Can the following Datalog programs be encoded using a C2RPQ? In each case, give a suitable C2RPQ or explain why there is none.

1. The "Same generation" Datalog program from the lecture:

$$\mathrm{S}(x, x) \leftarrow \mathrm{human}(x)$$
$$\mathrm{S}(x, y) \leftarrow \mathrm{parent}(x, w) \wedge \mathrm{S}(v, w) \wedge \mathrm{parent}(y, v)$$

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2. Ancestors born in the same city:

$$\begin{aligned} \mathrm{AncCity}(x, y, x', y') &\leftarrow \mathrm{parent}(x, x') \wedge \mathrm{bornIn}(x, y) \wedge \mathrm{bornIn}(x', y') \\ \mathrm{AncCity}(x, y, x'', y'') &\leftarrow \mathrm{AncCity}(x, y, x', y') \wedge \mathrm{AncCity}(x', y', x'', y'') \\ \mathrm{Query}(x, x', y) &\leftarrow \mathrm{AncCity}(x, y, x', y) \end{aligned}$$

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$$(\mathsf{parent} \circ \mathsf{parent}^*)(x, x') \wedge \mathsf{bornIn}(x, y) \wedge \mathsf{bornIn}(x', y)$$

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3. Ancestors of Dresden-based family lines:

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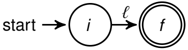

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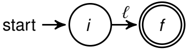

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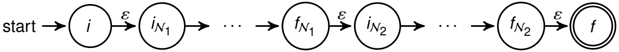

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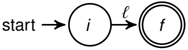

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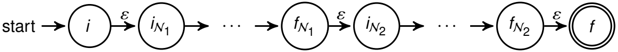
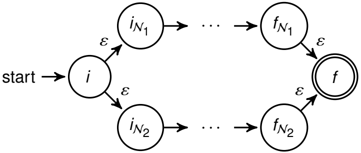


  - if $E = (E_1 + E_2)$, and $\mathcal{N}_1 \mathit{and} \mathcal{N}_2$ are NFAs deciding E_1 and E_2, then $\mathcal{N}$ is the following NFA:



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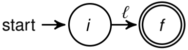

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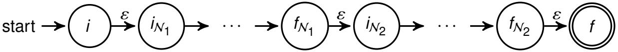

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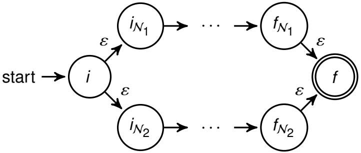
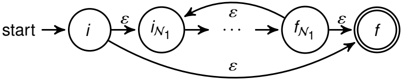


  - If $E = E_1^*$ and $\mathcal{N}_1$ is an NFA deciding E_1, then $\mathcal{N}$ is the following NFA:



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Solution.

- Let E, E' be regular expressions.
- Construct NFAs $\mathcal{N}$ and $\mathcal{N}'$ deciding $\mathcal{L}(E)$ and $\mathcal{L}(E')$.
- Use the powerset construction to obtain equivalent (but exponentially large) DFAs $\mathcal{D}$ and $\mathcal{D}'$.

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- $E \sqsubseteq E'$ iff $\mathcal{L}(\hat{\mathcal{D}})$ is empty: if there is $w \in \mathcal{L}(\hat{\mathcal{D}})$, then $w \in \mathcal{L}(E)$ but $w \notin \mathcal{L}(E')$.

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- Reachability on directed graphs can be checked in nondeterministic logarithmic space.

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- Let $\overline{\mathcal{D}'}$ be the DFA obtained from $\mathcal{D}'$ by making all *accepting* states *reject*, and vice versa. Then $w \in \overline{\mathcal{D}'}$ iff $w \notin \mathcal{D}'$.
- Construct the (polynomially large) product automaton $\hat{\mathcal{D}}$ of $\mathcal{D}$ and $\overline{\mathcal{D}'}$; then $\hat{\mathcal{D}}$ decides $E \cap \overline{E'}$.
- $E \sqsubseteq E'$ iff $\mathcal{L}(\hat{\mathcal{D}})$ is empty: if there is $w \in \mathcal{L}(\hat{\mathcal{D}})$, then $w \in \mathcal{L}(E)$ but $w \notin \mathcal{L}(E')$.
- $\mathcal{L}(\hat{\mathcal{D}})$ is empty iff the final state is not reachable from the initial state.
- Reachability on directed graphs can be checked in nondeterministic logarithmic space.
- Since the state graph of $\hat{\mathcal{D}}$ is exponentially large, we can decide emptiness in nondeterministic polynomial space.

Exercise 3.

Exercise. Consider the method for checking RPQ containment as sketched on slide “Containment for RPQs” in the lecture. Explain the procedure and the resulting complexity bounds in your own words. How could one construct the required automaton “on the fly”?

Solution.

- Let E, E' be regular expressions.
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- Since the state graph of $\hat{\mathcal{D}}$ is exponentially large, we can decide emptiness in nondeterministic polynomial space.
- Because of Savitch’s Theorem, we can thus decide containment in $\textsc{PSpace}$.

Exercise 4.

Exercise. Give an example for a binary C2RPQ that cannot be expressed as a 2RPQ.
By a *binary linear C2RPQ* we mean a C2RPQ of the form

$$\exists x_{k_1}, \ldots, x_{k_m}.\, \mathrm{R}_1(x_1, x_2) \wedge \mathrm{R}_2(x_2, x_3) \wedge \cdots \wedge \mathrm{R}_{n-1}(x_{n-1}, x_n)$$

where each $\mathrm{R}_i(x_i, x_{i+1})$ is an atom or a 2RPQ, and the x_{k_j} are among the variables that occur in the query. Can every linear binary C2RPQ be expressed by a 2RPQ? Explain your answer.

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$$\exists x.\ \mathsf{a}(x, x) \wedge \mathsf{b}(x, x),$$

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- Every atom $p(x_i, x_{i+1})$ in the query can be viewed as an RPQ with label p.

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- Since every 2RPQ in the query starts at the endpoint of the previous 2RPQ, the conjunctions can be replaced by composition.

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- Thus, $\exists x_2, \ldots, x_{n-1}.\ (\mathrm{R}_1 \circ \mathrm{R}_2 \circ \cdots \circ \mathrm{R}_{n-1})(x_1, x_n)$ is an equivalent 2RPQ.

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- Thus, $\exists x_2, \ldots, x_{n-1}.\, (\mathrm{R}_1 \circ \mathrm{R}_2 \circ \cdots \circ \mathrm{R}_{n-1})(x_1, x_n)$ is an equivalent 2RPQ.
- But in a 2RPQ, we lose access to $x_2, \ldots, x_{n-1}$.

Exercise 5.

Exercise. Give an example of a Datalog query that contains both of the following (and maybe also other) rules

$$\begin{aligned} \mathrm{Query}(x, z) &\leftarrow \mathrm{p}_a(x, y) \wedge \mathrm{p}_b(y, z) \\ \mathrm{Query}(x, z) &\leftarrow \mathrm{p}_a(x, x') \wedge \mathrm{Query}(x', z') \wedge \mathrm{p}_b(z', z) \end{aligned}$$

and that can be expressed as a C2RPQ.

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Solution.

- The query would match paths of the form $a^n b^n$ with $n \geq 0$, which is not a regular language.

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- The query would match paths of the form $a^n b^n$ with $n \geq 0$, which is not a regular language.
- We add rules so that all paths of the form $a^n b^m$ with $n, m \geq 0$ match, which is a regular language:

$$\begin{aligned}\mathrm{p}_{(a+b)^*}(x,y) &\leftarrow \mathrm{p}_a(x,y) & \mathrm{p}_{(a+b)^*}(x,y) &\leftarrow \mathrm{p}_b(x,y)\\ \mathrm{p}_{(a+b)^*}(x,y) &\leftarrow \mathrm{p}_{(a+b)^*}(x,z) \wedge \mathrm{p}_a(z,y) & \mathrm{p}_{(a+b)^*}(x,y) &\leftarrow \mathrm{p}_{(a+b)^*}(x,z) \wedge \mathrm{p}_b(z,y)\\ \mathrm{Query}(x,y) &\leftarrow \mathrm{p}_{(a+b)^*}(x,y)\end{aligned}$$

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$$\mathrm{Query}(x,y) \leftarrow \mathrm{p}_{(a+b)^*}(x,y)$$

- The resulting program is equivalent to the C2RPQ

$$(a+b)^*(x,y)$$