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Winter Term 2017/2018

Complexity Theory

**Exercise 3: Time Complexity**

14 November 2017

**Exercise 3.1.** A language $\mathbf{L} \in \mathrm{P}$ is complete for $\mathrm{P}$ under polynomial-time reductions if $\mathbf{L}' \leq_p \mathbf{L}$ for every $\mathbf{L}' \in \mathrm{P}$. Show that every language in $\mathrm{P}$ except $\emptyset$ and $\Sigma^*$ is complete for $\mathrm{P}$ under polynomial-time reductions.

**Exercise 3.2.** Let

$$\mathsf{A}_{\mathsf{PNTM}} = \{\langle \mathcal{M}, p, w\rangle \mid \mathcal{M} \text{ is a non-deterministic TM that accepts } w \text{ in time } p(|w|) \text{ with p a polynomial function}\}$$

Show that $\mathsf{A}_{\mathsf{PNTM}}$ is $\mathrm{NP}$-complete.

**Exercise 3.3.** Show that the following problem is $\mathrm{NP}$-complete:

Input: A propositional formula $\varphi$ in CNF
Question: Does $\varphi$ have at least 2 different satisfying assignments?

**Exercise 3.4.** We recall some definitions.

- Given some language $\mathbf{L}$, $\mathbf{L} \in \mathrm{coNP}$ if and only if $\overline{\mathbf{L}} \in \mathrm{NP}$.
- $\mathbf{L}$ is $\mathrm{coNP}$-hard if and only if $\mathbf{L}' \leq_p \mathbf{L}$ for every $\mathbf{L}' \in \mathrm{coNP}$.
- $\mathbf{L}$ is $\mathrm{coNP}$-complete if and only if $\mathbf{L} \in \mathrm{coNP}$ and $\mathbf{L}$ is $\mathrm{coNP}$-hard.

Show that if any $\mathrm{coNP}$-complete problem is in $\mathrm{NP}$, then $\mathrm{NP} = \mathrm{coNP}$.

**Exercise 3.5.** If $G$ is an undirected graph, a *vertex cover* of $G$ is a subset of the nodes where every edge of $G$ touches one of those nodes. The vertex cover problem asks whether a graph contains a vertex cover of a specified size.

$$\textsc{Vertex-Cover} = \{\langle G, k\rangle \mid G \text{ is an undirected graph that has a } k\text{-node vertex cover.}\}$$

Show that **Vertex-Cover** is $\mathrm{NP}$-complete.

**Hint:**
Try to find a reduction from 3-Sat.