Foundations of Knowledge Representation
Approximation Fixpoint Theory Problems

Problem 1. *Compute the least fixpoint of the operator* ${}_{P_1}T$ *for the definite logic program* P_1 *defined as follows:*

$P_1 =$

$$
\begin{array}{ll}
p \leftarrow q, r & p \leftarrow s, t \\
w \leftarrow s, r & z \leftarrow p, w \\
s \leftarrow p, q & v \leftarrow w \\
r \leftarrow t, m & q \leftarrow \\
t \leftarrow & m \leftarrow
\end{array}
$$

Problem 2. *Approximation Fixpoint Theory can be used to define the semantics of argumentation frameworks. For an AF* $F = (A, R)$ *we thus consider the complete lattice* $(L, \subseteq)$ *with* $L = 2^A$ *and its associated bilattice* $(L^2, \leq_i)$.

The stable extension semantics is characterised by the operator ${}_FU$ *defined as follows:*

Definition 1. *For each AF* $F = (A, R)$*, the operator* ${}_FU : A \to A$ *yields – for a given set* $S \subseteq A$ *– a new set*

$$
{}_FU(S) = \{a \in A \mid b \in S \text{ implies } (b, a) \notin R\}
$$

Intuitively, the operator returns the set of all arguments that are unattacked (hence the U*) by the input set.*

An approximator of ${}_FU$ *is given by* ${}_F\mathcal{U}$ *defined as follows:*

Definition 2. *For each AF* $F = (A, R)$*, the operator* ${}_F\mathcal{U} : 2^A \times 2^A \to 2^A \times 2^A$ *yields – for a given pair* $(P, S) \in 2^A \times 2^A$ *– a new pair*

$$
{}_F\mathcal{U}(P, S) = ({}_FU(S), {}_FU(P))
$$

Now, consider the two AFs: $F_1 = (A_1, R_1)$ *with* $A_1 = \{a, b\}$ *and* $R_1 = \{a, b, (b, a)\}$ *and* $F_2 = (A_2, R_2)$ *with* $A_2 = \{a, b, c\}$ *and* $R_2 = \{(a, b), (b, c), (c, a)\}$.

Do the following:

(i) For both frameworks F_i, $i = 1, 2$, illustrate the corresponding complete lattice $(2^{A_i}, \subseteq)$) and the mappings of ${}_{F_i}U$.

(ii) Show that ${}_FU$ characterises stable extension semantics. More precisely: Show that for every AF F, every set $S \subseteq A$ is a stable extension of F iff ${}_FU(S) = S$.

(iii) Argue whether or not ${}_FU$ is $\subseteq$-monotone for every AF F.

(iv) Show that ${}_F\mathcal{U}$ is an approximator for ${}_FU$.

Problem 3. *Consider the normal logic programs P_2 and P_3:*

$P_2 =$

$$\begin{array}{ll} p \leftarrow p,\, q & \qquad q \leftarrow p \\ r \leftarrow {\sim} s & \qquad s \leftarrow {\sim} q \end{array}$$

$P_3 =$

$$\begin{array}{ll} p \leftarrow q,\, s & \\ q \leftarrow r,\, {\sim} u & \\ r \leftarrow u & \qquad r \leftarrow \\ s \leftarrow {\sim} t & \\ t \leftarrow u & \end{array}$$

Recall the definition of the approximator ${}_P\mathcal{T}$ from the lecture. Compute the least fixpoint of ${}_{P_2}\mathcal{T}$ and ${}_{P_3}\mathcal{T}$.