TECHNISCHE UNIVERSITÄT DRESDEN

Artificial Intelligence, Computational Logic

ABSTRACT ARGUMENTATION

Equivalences of Argumentation Frameworks

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Dresden, 16th September 2015

Motivation

- Argumentation is a dynamic reasoning process.
- During the process the participants come up with new arguments.
    - Which effects causes additional information wrt. a semantics?
    - Which information does not contribute to the results?

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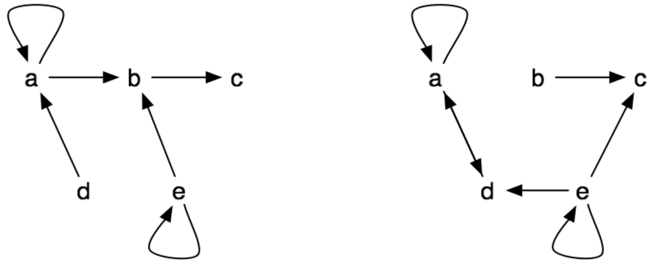
  - One can savely replace an AF by a strongly equivalent one without changing its extensions.
- In a negotiation between two agents: SE allows to characterize situations where the two agents have an equivalent view of the world which is moreover robust to additional information.

Outline

1. Standard Equivalence
2. Strong Equivalence
3. Other Notions of Equivalence
4. Summary

Standard Equivalence of AFs

Example



- AFs F and G are equivalent (wrt. stable semantics).
- $stable(F) = stable(G) = \emptyset$

Standard Equivalence

Intuitively, semantically equivalent AF yield the same result when applying the semantic operators.

Standard Equivalence [Oikarinen and Woltran, 2011]

AFs F and G are standard equivalent wrt. a given semantic σ ($F \equiv^{\sigma} G$), iff they posses the same extensions under the semantic σ.

Results wrt. Standard Equivalence

For any AFs F and G we have

- $adm(F) = adm(G) \implies pref(F) = pref(G)$
- $adm(F) = adm(G) \implies ideal(F) = ideal(G)$
- $comp(F) = comp(G) \implies pref(F) = pref(G)$
- $comp(F) = comp(G) \implies ground(F) = ground(G)$
- $comp(F) = comp(G) \implies ideal(F) = ideal(G)$
- $adm(F) = adm(G)$ and $semi(F) = semi(G) \implies eager(F) = eager(G)$

Standard Equivalence of AFs

Intuitively, semantically equivalent AF yield the same result when applying the semantic operators.

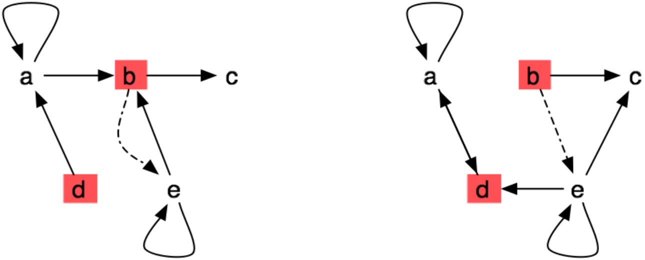
Standard Equivalence

AFs F and G are standard equivalent wrt. a given semantic σ ($F \equiv^\sigma G$), iff they posses the same extensions under the semantic σ.

- Appropriate from a static view point.
- But AA is not static, rather a highly dynamic process, they are expanded over time (e.g., during an analysis phase)

Strong Equivalence of AFs

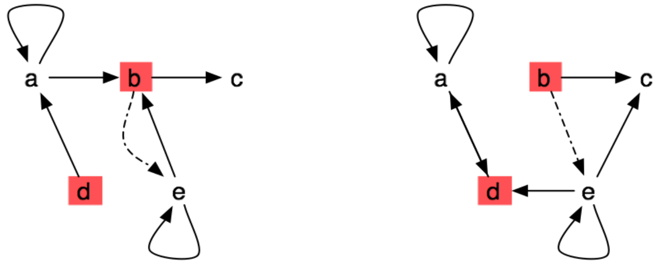
Example



- $stable(F \cup H) = stable(G \cup H) = \{\{b, d\}\}$.

Strong Equivalence of AFs

Example



- Goal: identify redundant attacks:
  - Find attacks which do not contribute in the evaluation of F, no matter how F is extended

$\Longrightarrow$ Define kernel of an AF (remove redundant attacks)

$\Longrightarrow$ Checking for strong equivalence reduces to check syntactic equivalence

Motivation ctd.

- Identification of redundant attacks is important in choosing an appropriate semantics.
- Caminada and Amgoud outlined in [Caminada and Amgoud, 2007] that the interplay between how a framework is built and which semantics is used to evaluate the framework is crucial in order to obtain useful results when the (claims of the) arguments selected by the chosen semantics are collected together.
- Knowledge about redundant attacks (wrt. a particular semantics) might help to identify unsuitable such combinations.
- Strong equivalence has been analyzed for many semantics in [Oikarinen and Woltran, 2010].
- Naive-based semantics naive, stage and *cf2* have been analyzed in [Gaggl and Woltran, 2013].

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Strong Equivalence (SE)

Respecting dynamic aspects, one needs to develop stronger equivalent notions.

Strong Equivalence [Oikarinen and Woltran, 2010]

Two AFs F and G are strongly equivalent to each other wrt. a semantics σ, in symbols $F \equiv_s^\sigma G$, iff for each AF H, $\sigma(F \cup H) = \sigma(G \cup H)$.

- By definition $F \equiv_s^\sigma G$ implies $\sigma(F) = \sigma(G)$
- The AF H represents possible (dynamic) growth of F and G

Outline

1 Standard Equivalence

2 Strong Equivalence
   Identification of Kernels
   SE wrt. Naive-based Semantics

3 Other Notions of Equivalence

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SE wrt. Stable Semantics

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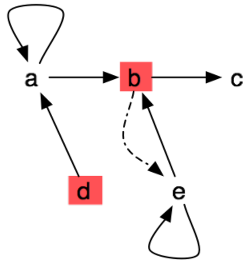

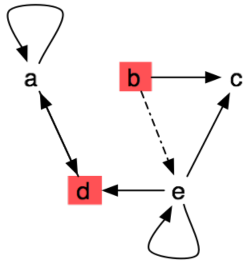

s-kernel

For an AF $F = (A, R)$ we define the s-kernel of F as $F^{sk} = (A, R^{sk})$ where

$$R^{sk} = R \setminus \{(a, b) \mid a \neq b, (a, a) \in R\}.$$

SE wrt. Stable Semantics

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SE wrt. Stable Semantics

- For any AF F, $stable(F) = stable(F^{sk})$
- Let F and G be AFs, s.t. $F^{sk} = G^{sk}$. Then, $(F \cup H)^{sk} = (G \cup H)^{sk}$ for each AF H
- For any AFs F and G: $F^{sk} = G^{sk}$ iff $F \equiv_s^{stable} G$

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SE wrt. Stage Semantics

- For any AFs F and G: $F^{sk} = G^{sk}$ iff $F \equiv_s^{stage} G$

SE wrt. Admissible Semantics

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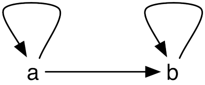

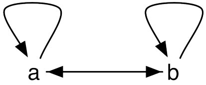

SE wrt. Admissible Semantics

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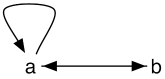

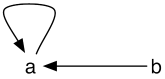

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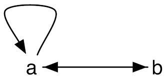

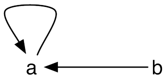

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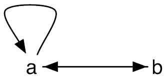

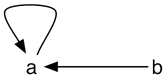

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SE wrt. Admissible Semantics

- For any AF F and G, $F^{ak} = G^{ak} \implies F^{sk} = G^{sk}$
- For any AF F, $\sigma(F) = \sigma(F^{ak})$ for $\sigma \in \{adm, pref, ideal, semi, eager\}$
- If $F^{ak} = G^{ak}$, then $(F \cup H)^{ak} = (G \cup H)^{ak}$ for each AF H
- For any AFs F and G: $F^{ak} = G^{ak}$ iff $F \equiv_s^\sigma G$ for $\sigma \in \{adm, pref, ideal, semi, eager\}$

SE wrt. Grounded Semantics

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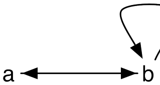

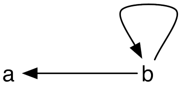

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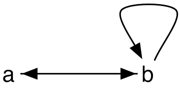

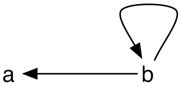

g-kernel

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SE wrt. Grounded Semantics

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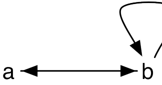

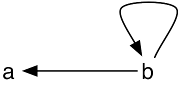

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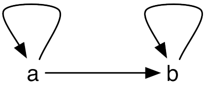
SE wrt. Grounded Semantics

- For any AF F, $ground(F) = ground(F^{gk})$
- Let F and G be AFs, s.t. $F^{gk} = G^{gk}$. Then, $(F \cup H)^{gk} = (G \cup H)^{gk}$ for each AF H
- For any AFs F and G: $F^{gk} = G^{gk}$ iff $F \equiv_s^{ground} G$

SE wrt. Complete Semantics

Example



c-kernel

For an AF $F = (A, R)$ we define the c-kernel of F as $F^{ck} = (A, R^{gk})$ where

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SE wrt. Complete Semantics

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SE wrt. Complete Semantics

- For any AFs F and G, $F^{ck} = G^{ck} \implies F^{\tau} = G^{\tau}$ for $\tau \in \{sk, ak, gk\}$
- Let F and G be AFs, s.t. $F^{ck} = G^{ck}$ iff jointly $F^{ak} = G^{ak}$ and $F^{gk} = G^{gk}$
- For any AF F, $comp(F) = comp(F^{ck})$
- Let F and G be AFs, s.t. $F^{ck} = G^{ck}$. Then, $(F \cup H)^{ck} = (G \cup H)^{ck}$ for each AF H
- For any AFs F and G: $F^{ck} = G^{ck}$ iff $F \equiv_s^{comp} G$

SE and Self-Loops

Self-Loop Free AFs

For any self-loop free AF F,

$$F = F^{sk} = F^{ak} = F^{ck} = F^{gk}$$

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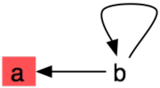
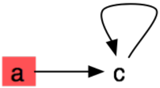
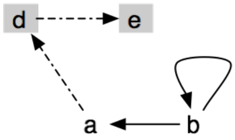
SE wrt. *naive* Semantics

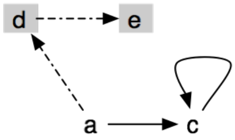




- $naive(F) = naive(G) = \{\{a\}\}$

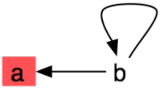
SE wrt. *naive* Semantics

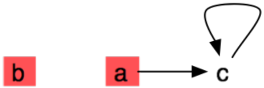




- $naive(F \cup H) = naive(G \cup H) = \{\{d\}, \{a, e\}\}$

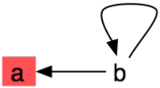
SE wrt. *naive* Semantics

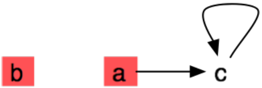




- $naive(F \cup H) = naive(F) = \{\{a\}\}$ but
- $naive(G \cup H) = \{\{a, b\}\}$.

SE wrt. *naive* Semantics





- $naive(F \cup H) = naive(F) = \{\{a\}\}$ but
- $naive(G \cup H) = \{\{a, b\}\}$.

Theorem ([Gaggl and Woltran, 2013])

The following statements are equivalent:

1. $F \equiv_s^{naive} G$;
2. $naive(F) = naive(G)$ and $A(F) = A(G)$;
3. $cf(F) = cf(G)$ and $A(F) = A(G)$.

Strong Equivalence wrt. *cf2*

Theorem ([Gaggl and Woltran, 2013])

For any AFs F and G, $F \equiv_s^{cf2} G$ iff $F = G$.

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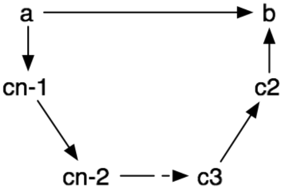

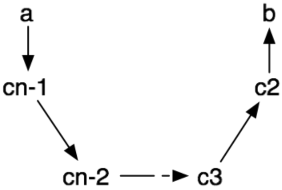

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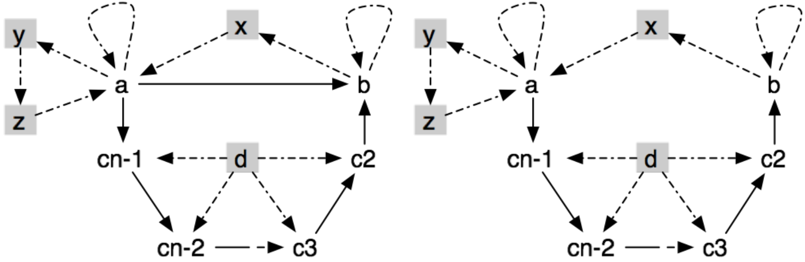

$$H = (A \cup \{d, x, y, z\}, \{(a,a),(b,b),(b,x),(x,a),(a,y),(y,z),(z,a),(d,c) \mid c \in A \setminus \{a,b\}\}).$$

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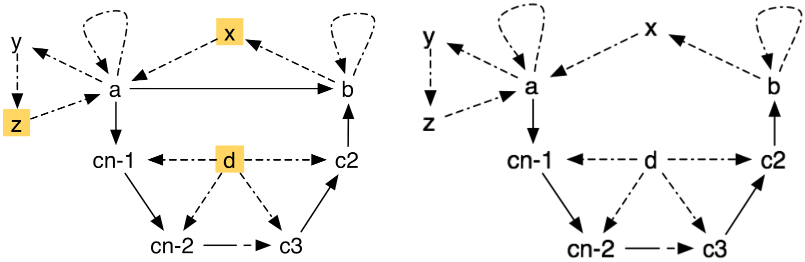

Let $E = \{d, x, z\}$, $E \in cf2(F \cup H)$ but $E \notin cf2(G \cup H)$.

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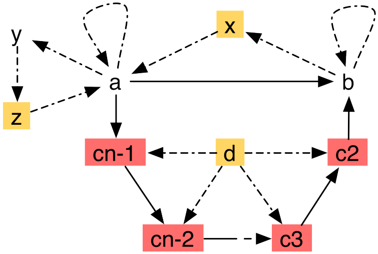

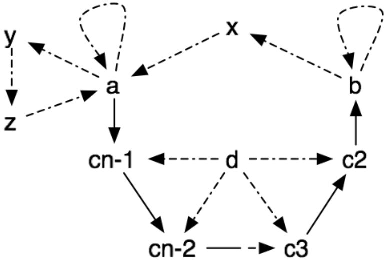

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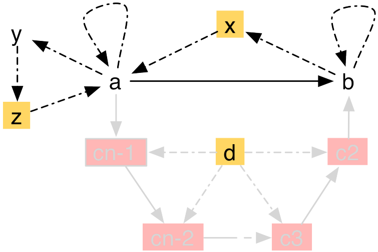

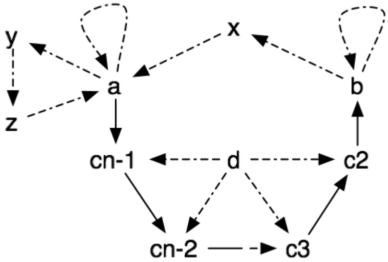

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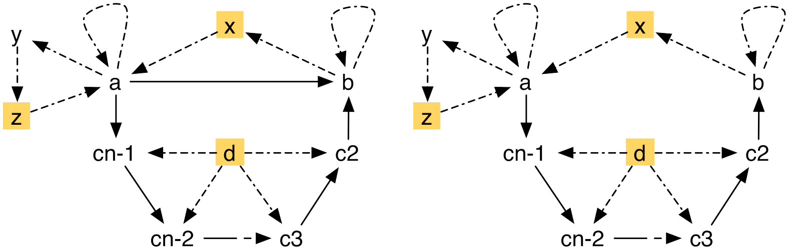

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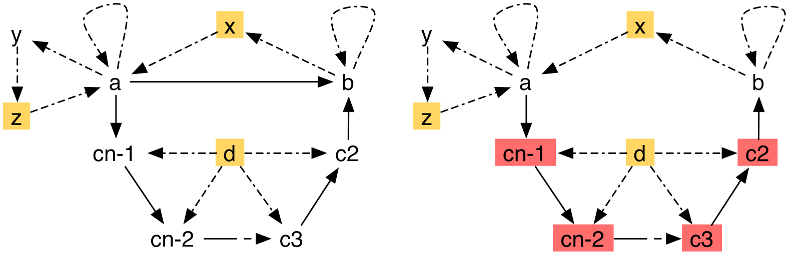

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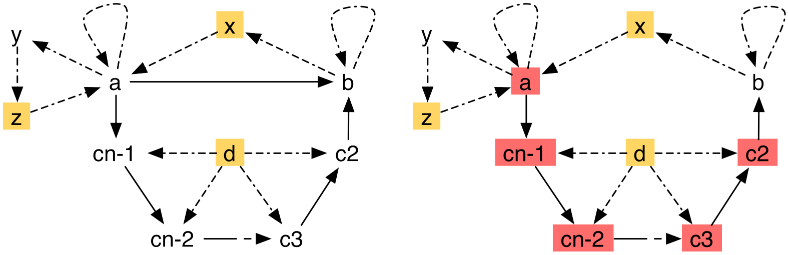
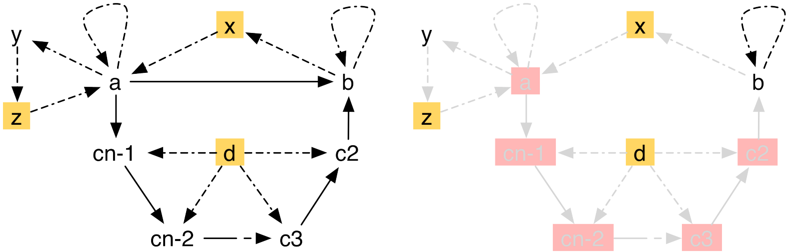


Let $E = \{d, x, z\}$, $E \in cf2(F \cup H)$ but $E \notin cf2(G \cup H)$.

Strong Equivalence wrt. *cf2*

Theorem ([Gaggl and Woltran, 2013])

For any AFs F and G, $F \equiv_s^{cf2} G$ iff $F = G$.



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SE wrt. *cf2*

Theorem

For any AFs F and G, $F \equiv_s^{cf2} G$ iff $F = G$.

- No matter which AFs $F \neq G$, one can always construct an H s.t. $cf2(F \cup H) \neq cf2(G \cup H)$;

SE wrt. *cf2*

Theorem

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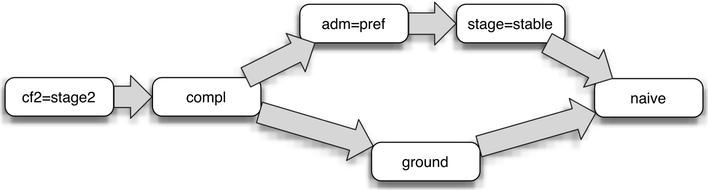
- No matter which AFs $F \neq G$, one can always construct an H s.t. $cf2(F \cup H) \neq cf2(G \cup H)$;

Succinctness Property [Gaggl and Woltran, 2013]

An argumentation semantics σ satisfies the succinctness property or is maximal succinct iff no AF contains a redundant attack wrt. σ.

Comparing Semantics wrt. SE



Outline

1. Standard Equivalence
2. Strong Equivalence
3. Other Notions of Equivalence
4. Summary

Expansion of an AF

[Baumann and Brewka, 2013, Baumann, 2012]

Back to H ...

- H might be of certain nature, s.t. $F \cup H$ can be characterized,
- which in turn yields different (strong) equivalences.

Expansion

An AF F^* is an expansion of AF $F = (A, R)$ (for short $F \preceq_E F^*$), iff $F^* = (A \cup A^*, R \cup R^*)$ where $A \cap A^* = R \cap R^* = \emptyset$. An expansion is

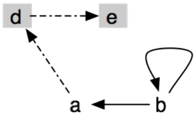
1. normal ($F \preceq_N F^*$), iff $\forall a, b\ ((a, b) \in R^* \rightarrow a \in A^* \vee b \in A^*)$,
2. strong ($F \preceq_S F^*$), iff $F \preceq_N F^*$ and $\forall a, b\ ((a, b) \in R^* \rightarrow \neg(a \in A \wedge b \in A^*))$,
3. weak ($F \preceq_W F^*$), iff $F \preceq_N F^*$ and $\forall a, b\ ((a, b) \in R^* \rightarrow \neg(a \in A^* \wedge b \in A))$,
4. local ($F \preceq_L F^*$), iff $A^* = \emptyset$.

Expansions of an AF

Expansion

An AF F^* is an expansion of AF $F = (A, R)$ (for short $F \preceq_E F^*$), iff $F^* = (A \cup A^*, R \cup R^*)$ where $A \cap A^* = R \cap R^* = \emptyset$. An expansion is

1. normal ($F \preceq_N F^*$), iff $\forall a, b\ ((a, b) \in R^* \rightarrow a \in A^* \vee b \in A^*)$,
2. strong ($F \preceq_S F^*$), iff $F \preceq_N F^*$ and $\forall a, b\ ((a, b) \in R^* \rightarrow \neg(a \in A \wedge b \in A^*))$,
3. weak ($F \preceq_W F^*$), iff $F \preceq_N F^*$ and $\forall a, b\ ((a, b) \in R^* \rightarrow \neg(a \in A^* \wedge b \in A))$,
4. local ($F \preceq_L F^*$), iff $A^* = \emptyset$.



- F^* is a weak expansion
- F^* is NOT strong or local

Notions of Equivalence

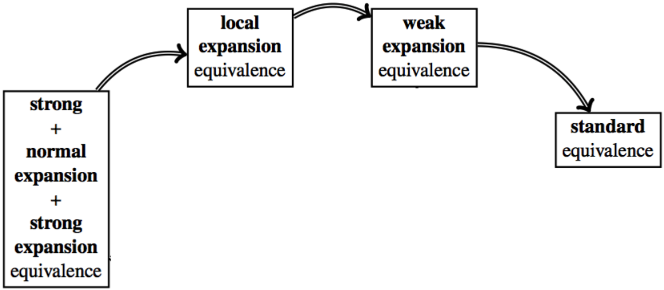
Equivalence relations are now developed wrt. the type of expansion.

Notions of Equivalence

Given a semantic σ. Two AFs F and G are

- normal expansion equivalent wrt. σ ($F \equiv_N^\sigma G$) iff for each AF H, s.t. $F \preceq_N F \cup H$ and $G \preceq_N G \cup H$, $F \cup H \equiv^\sigma G \cup H$ holds,
- strong expansion equivalent wrt. σ ($F \equiv_S^\sigma G$) iff for each AF H, s.t. $F \preceq_S F \cup H$ and $G \preceq_S G \cup H$, $F \cup H \equiv^\sigma G \cup H$ holds,
- weak expansion equivalent wrt. σ ($F \equiv_W^\sigma G$) iff for each AF H, s.t. $F \preceq_W F \cup H$ and $G \preceq_W G \cup H$, $F \equiv^\sigma G \cup H$ holds,
- local expansion equivalent wrt. σ ($F \equiv_L^\sigma G$) iff for each AF G, s.t. $A(H) \subseteq A(F \cup G)$, $F \cup H \equiv^\sigma G \cup H$ holds.

Relations for Stable Semantics



Outline

1. Standard Equivalence
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Summary

- We identified kernels for stable, admissible ($pref$, $ideal$, $semi$, $eager$), complete and grounded semantics
- We provide characterizations for strong equivalence wrt. stage, naive and *cf2* semantics.
- *cf2* semantics is the only one where no redundant attacks exist.
- *cf2* semantics treats self-loops in a more sensitive way than other semantics.
- More fine grained characterization of equivalence wrt. expansions

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