TECHNISCHE UNIVERSITÄT DRESDEN

DATABASE THEORY

Lecture 7: Tree-Like Conjunctive Queries

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Knowledge-Based Systems

TU Dresden, 14th May 2018

Review

Conjunctive queries (CQs) are simpler than FO-queries:

- NP combined and query complexity (instead of PSpace)
- data complexity remains in AC^0

CQs become even simpler if they are tree-shaped:

- GYO algorithm defines acyclic hypergraphs
- acyclic hypergraphs have join trees
- join trees can be evaluated in P with Yannakakis' Algorithm

This time:

- Find more general conditions that make CQs tractable
  $\leadsto$ "tree-like" queries that that are not really trees
- Play some games

Is Yannakakis' Algorithm Optimal?

We saw that tree queries can be evaluated in polynomial time,
but we know that there are much simpler complexity classes:

$$\mathsf{NC}^0 \subset \mathsf{AC}^0 \subset \mathsf{NC}^1 \subseteq \mathsf{L} \subseteq \mathsf{NL} \subseteq \mathsf{AC}^1 \subseteq \ldots \subseteq \mathsf{NC} \subseteq \mathsf{P}$$

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Indeed, tighter bounds have been shown:

Theorem 7.1 (Gottlob, Leone, Scarcello: J. ACM 2001): Answering tree BCQs is complete for LOGCFL.

LOGCFL: the class of problems LogSpace-reducible to the word problem of a context-free language:

$$\mathrm{NC}^0 \subset \mathrm{AC}^0 \subset \mathrm{NC}^1 \subseteq \mathrm{L} \subseteq \mathrm{NL} \subseteq \mathrm{LOGCFL} \subseteq \mathrm{AC}^1 \subseteq \ldots \subseteq \mathrm{NC} \subseteq \mathrm{P}$$

$\leadsto$ highly parallelisable

Generalising Tree Queries

In practice, many queries are tree queries,
but even more queries are "almost" tree queries, but not quite ...

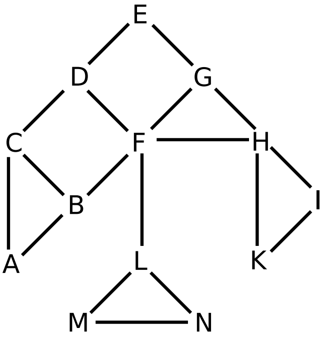
How can we formalise this idea?

Several attempts to define "tree-like" queries:

- Treewidth: a way to measure tree-likeness of graphs
- Query width: towards tree-like query graphs
- Hypertree width: adoption of treewidth to hypergraphs

How to recognise trees . . .

. . . from quite a long way away:



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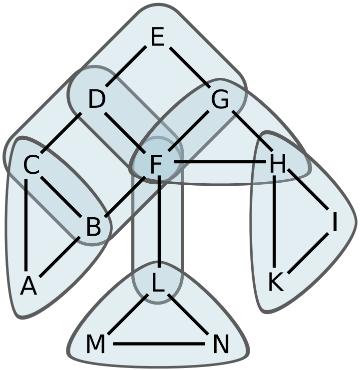


Tree Decompositions

Idea: if we can group the edges of a graph into bigger pieces, these pieces might form a tree structure

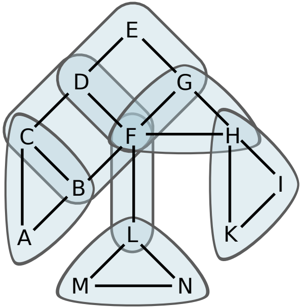
Definition 7.2: Consider a graph $G = \langle V, E \rangle$. A tree decomposition of G is a tree structure T where each node of T is a subset of V, such that:

- The union of all nodes of T is V.
- For each edge $(v_1 \to v_2) \in E$, there is a node N in T such that $v_1, v_2 \in N$.
- For every vertex $v \in V$, the set of nodes of T that contain v form a subtree of T; equivalently: if two nodes contain v, then all nodes on the path between them also contain v (connectedness condition).

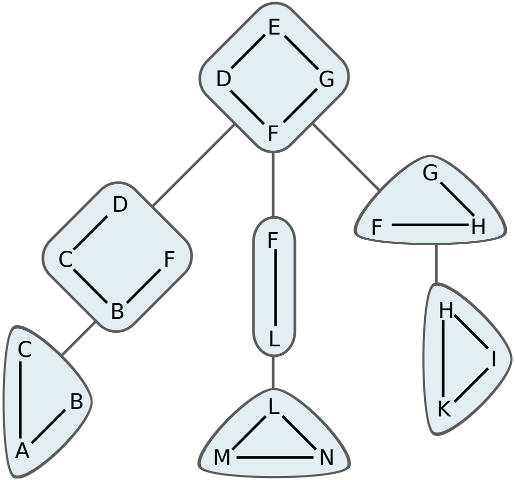
Nodes of a tree decomposition are often called bags

(not related to the common use of “bag” as a synonym for “multiset”)

Tree Decompositions: Example



Tree Decompositions: Example



Treewidth

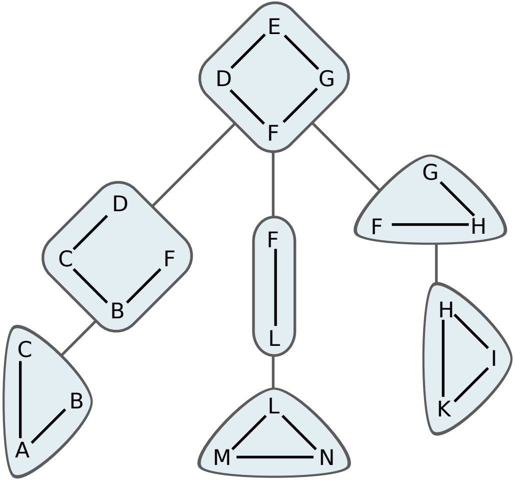
The treewidth of a graph defines how “tree-like” it is:

Definition 7.3: The width of a tree decomposition is the size of its largest bag minus one.
The treewidth of a graph G, denoted $\mathrm{tw}(G)$, is the smallest width of any of its tree decompositions.

Simple observations:

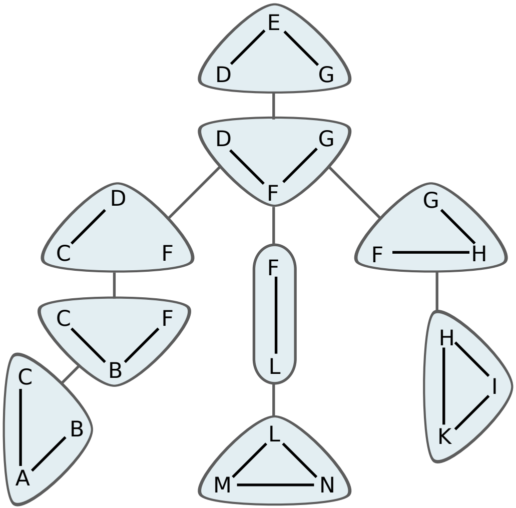
- If G is a tree, then we can decompose it into bags that contain only one edge
  $\leadsto$ trees have treewidth 1
- Every graph has at least one tree decomposition where all vertices are in one bag
  $\leadsto$ maximal treewidth = number of vertices -1

Treewdith: Example



$\rightsquigarrow$ tree decomposition of width 3

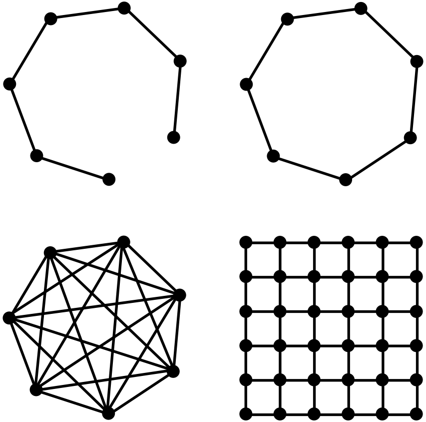
Treewdith: Example



$\rightsquigarrow$ tree decomposition of width 2 = treewidth of the example graph

More Examples

What is the treewidth of the following graphs?



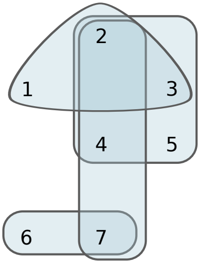
Treewidth and Conjunctive Queries

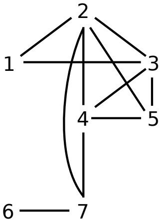
Treewidth is based on graphs, not hypergraphs

Treewidth and Conjunctive Queries

Treewidth is based on graphs, not hypergraphs
$\rightsquigarrow$ treewidth of CQ = treewidth of primal graph of query hypergraph

Query graph and corresponding primal graph:





$\rightsquigarrow$ Treewidth 3

Observation: acyclic hypergraphs can have unbounded treewidth!

Exploiting Treewidth in CQ Answering

Queries of low treewidth can be answered efficiently:

Theorem 7.4 (Dechter/Chekuri+Rajamaran '97/Kolaitis+Vardi '98/Gottlob & al. '98): Answering BCQs of treewidth k is possible in time $O(n^k \log n)$, and thus in polynomial time if k is fixed.
The problem is also complete for LOGCFL.

Checking for low treewidths can also be done efficiently:

Theorem 7.5 (Bodlaender '96): Given a graph G and a fixed number k, one can check in linear time if tw$(G) \leq k$, and the corresponding tree decomposition can also be found in linear time.

Warning: neither CQ answering nor tree decomposition might be practically feasible if k is big

Treewidth via Games

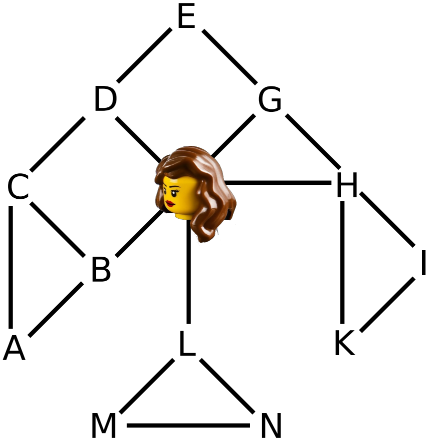
Seymour and Thomas [1993] gave
an alternative characterisation of treewidth:

The Cops-and-Robber Game

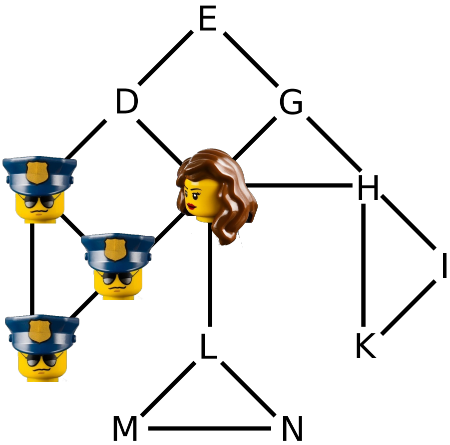
- The game is played on a graph G
- There are k cops and one robber, each located at one vertex
- In each turn:
  - the cops can fly to an arbitrary vertex in the graph
  - the robber can run along the edges of the graph, as far as she likes, as long a she does not pass through any vertex that was occupied by a cop before or after the turn

    (the robber can run to a place where a cop was before the turn, but not pass through such a place)
- The goal of the cops is to catch the robber;
  the goal of the robber is never to be caught

Cops and Robbers: Example



Cops and Robbers: Example



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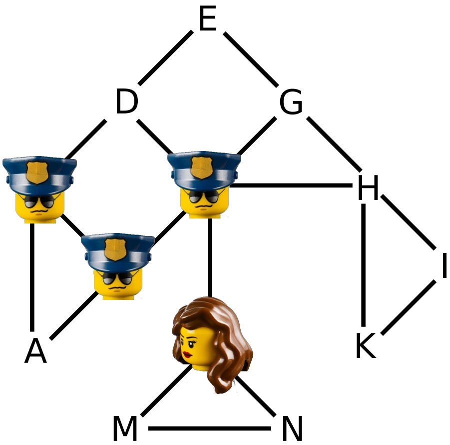

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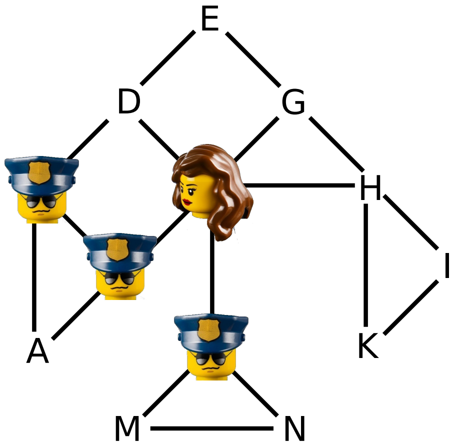

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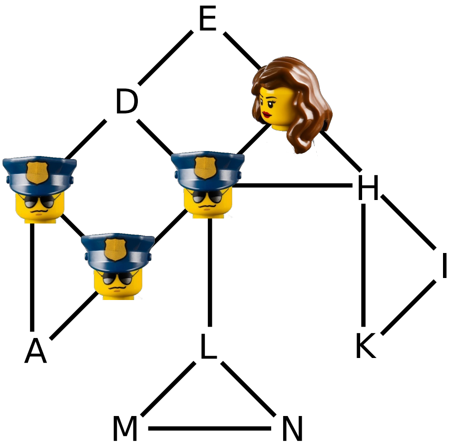

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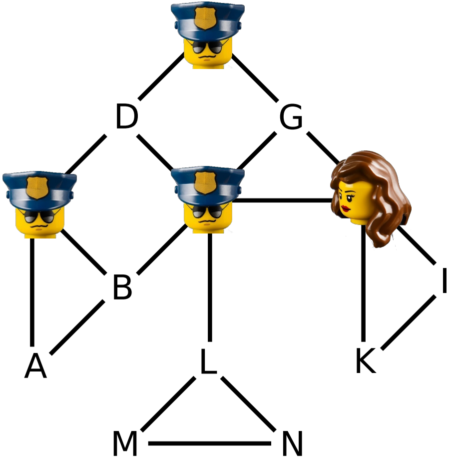

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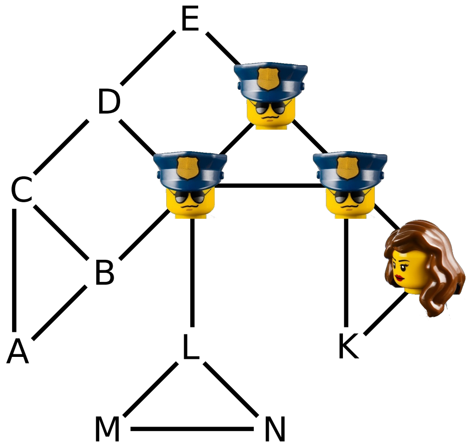

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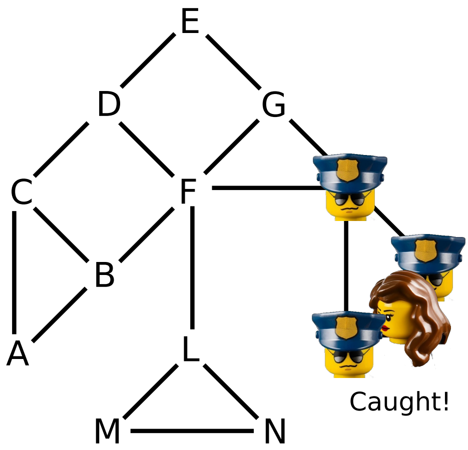

Cops & Robbers and Treewidth

Theorem 7.6 (Seymour and Thomas): A graph G is of treewidth $\leq k-1$ if and only if k cops have a winning strategy in the cops & robber game on G.

Intuition: the cops together can block even the widest branch and still move in on the robber

Treewidth via Logic

Kolaitis and Vardi [1998] gave a logical characterisation of treewidth

Bounded treewidth CQs correspond to certain FO-queries:

- We allow FO-queries with $\exists$ and $\wedge$ as only operators
- But operators can be nested in arbitrary ways (unlike in CQs)
- Theorem: A query can be expressed with a CQ of treewidth k if and only if it can be expressed in this logic using a query with at most $k + 1$ distinct variables

Intuition: variables can be reused by binding them in more than one $\exists$
$\leadsto$ Apply a kind of "inverted prenex-normal-form transformation"
$\leadsto$ Variables that occur in the same atom or in a "tightly connected" atom must use different names
$\leadsto$ minimum number of variables $\Leftrightarrow$ treewidth (+1)

Summary and Outlook

Treewidth has Pros and Cons:

Advantages:

- Bounded treewidth is easy to check
- Bounded treewidth CQs are easy to answer

Disadvantages:

- Even families of acyclic graphs may have unbounded treewidth
- Loss of information when using primal graph
  (cliques might be single hyperedges – linear! –
  or complex query patterns – exponential!)

Open questions:

- Are there better ways to capture “tree-like” queries?