Actions and Causality

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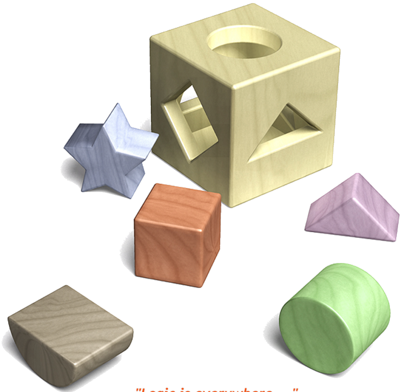
- Introduction
- Conjunctive Planning Problems
- The Fluent Calculus



"Logic is everywhere ..."

States, Actions, and Causality

- **Rational Agents, Agent Programming Languages, Cognitive Robotics**
- **Situation Calculus** McCarthy 1963
- **Core Ideas**
  - **A state is a snapshot of the world and**
  - **can only be changed by actions**
- **A state is specified with the help of fluents**
- **Problem** **Each state and each action is only partially known!**

General Problems

- **Frame problem**
  Which fluents are unaffected by the execution of an action?
- **Ramification problem**
  Which fluents are really present after the execution of an action?
- **Qualification problem**
  Which preconditions have to be satisfied such that an action is executable?
- **Prediction problem**
  How long are fluents present in certain situations?

- **All problems have a cognitive as well as a technical aspect**
- **Only the frame problem is considered in this lecture**

Requirements

- McCarthy 1963
- General properties of causality and facts about the possibility and results of actions are given as formulas
- It is a logical consequence of the facts of a state and the general axioms that goals can be achieved by performing certain actions
- The formal descriptions of states should correspond as closely as possible to what people may reasonably be presumed to know about them when deciding what to do

Conjunctive Planning Problems

- **Initial state** $\mathcal{I}$: $\dot{\{} i_1, \ldots, i_m \dot{\}}$ **of ground fluents**
- **Goal state** $\mathcal{G}$: $\dot{\{} g_1, \ldots, g_n \dot{\}}$ **of ground fluents**
- **Finite set $\mathcal{A}$ of actions of the form**

$$\dot{\{} c_1, \ldots, c_l \dot{\}} \Rightarrow \dot{\{} e_1, \ldots, e_k \dot{\}},$$

  where $\dot{\{} c_1, \ldots, c_l \dot{\}}$ and $\dot{\{} e_1, \ldots, e_k \dot{\}}$ are multisets of fluents called conditions and (direct) effects, respectively

- **Assumption**
  Each variable occurring in the effects of an action occurs also in its conditions
- **A conjunctive planning problem is the question of whether there exists a sequence of actions whose execution transforms the initia into the goal state**

Actions and Plans

- Let $\mathcal{S}$ be a multiset of ground fluents
- $\mathcal{C} \Rightarrow \mathcal{E}$ is applicable in $\mathcal{S}$ iff there exists θ such that $\mathcal{C}\theta \dot{\subseteq} \mathcal{S}$
- The application of $\mathcal{C} \Rightarrow \mathcal{E}$ in $\mathcal{S}$ leads to $\mathcal{S}' = (\mathcal{S} \dot{\setminus} \mathcal{C}\theta) \dot{\cup} \mathcal{E}\theta$
  - One should observe that $\mathcal{S}'$ is ground
    - $\mathcal{S}$ is ground
    - $var(\mathcal{E}) \subseteq var(\mathcal{C})$
    - θ is grounding
- A plan is a sequence of actions
- A goal $\mathcal{G}$ is satisfied
  iff there exists a plan p which transforms $\mathcal{I}$ into $\mathcal{S}$ and $\mathcal{G} \dot{\subseteq} \mathcal{S}$
  - Such a plan is called solution for the planning problem

Blocks World

- The **pickup** action

$$pickup(V): \quad \dot{\{} clear(V), ontable(V), empty \dot{\}} \Rightarrow \dot{\{} holding(V) \dot{\}}$$

- The **unstack** action

$$unstack(V, W): \quad \dot{\{} clear(V), on(V, W), empty \dot{\}} \Rightarrow \dot{\{} holding(V), clear(W) \dot{\}}$$

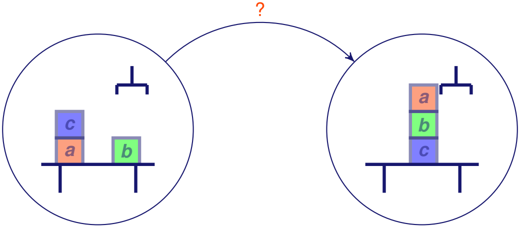
- The **putdown** action

$$putdown(V): \quad \dot{\{} holding(V) \dot{\}} \Rightarrow \dot{\{} clear(V), ontable(V), empty \dot{\}}$$

- The **stack** action

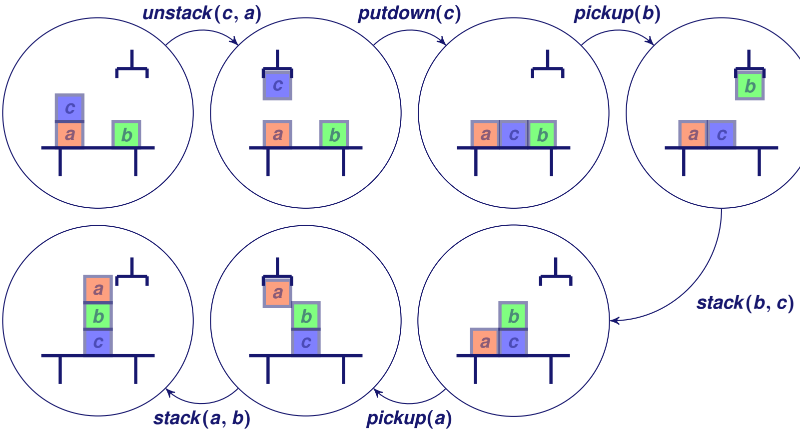
$$stack(V, W): \quad \dot{\{} holding(V), clear(W) \dot{\}} \Rightarrow \dot{\{} on(V, W), clear(V), empty \dot{\}}$$

Sussman's Anomaly



- $\mathcal{I} = \dot{\{}$*ontable*(a), *ontable*(b), *on*(c, a), *clear*(b), *clear*(c), *empty*$\dot{\}}$
- $\mathcal{G} = \dot{\{}$*ontable*(c), *on*(b, c), *on*(a, b), *clear*(a), *empty*$\dot{\}}$
- **Solution**
  [*unstack*(c, a), *putdown*(c), *pickup*(b), *stack*(b, c), *pickup*(a), *stack*(a, b)]
- **What happens if we independently search for shortest solutions for the two subgoals** *on*(a, b) **and** *on*(b, c)**?**

Sussman's Anomaly – Solution



A Fluent Calculus Implementation – Actions and Causality

- **An action $\mathcal{C} \Rightarrow \mathcal{E}$ is represented by $action(\mathcal{C}^{-I}, name, \mathcal{E}^{-I})$, where *name* is a term identifying the action**

$$
\begin{array}{l}
action(clear(V) \circ ontable(V) \circ empty,\; pickup(V),\; holding(V)) \\
action(clear(V) \circ on(V, W) \circ empty,\; unstack(V, W),\; holding(V) \circ clear(W)) \\
action(holding(V),\; putdown(V),\; clear(V) \circ ontable(V) \circ empty) \\
action(holding(V) \circ clear(W),\; stack(V, W),\; on(V, W) \circ clear(V) \circ empty)
\end{array}
$$

- **Causality is expressed by $causes(s, p, s')$, where s and s' are fluent terms and p is a list of actions representing a plan**

$$
\begin{array}{lcl}
causes(X, [\,], Y) & \leftarrow & X \approx Y \circ Z \\
causes(X, [V|W], Y) & \leftarrow & action(P, V, Q) \\
 & & \wedge\ P \circ Z \approx X \\
 & & \wedge\ causes(Z \circ Q, W, Y) \\
X \approx X & &
\end{array}
$$

A Fluent Calculus Implementation – The Planning Problem

- Let $\mathcal{K}_A$ be the set of facts representing actions
- Let $\mathcal{K}_C$ be the set of clauses representing causality
- The planning problem (with intial and goal state $\mathcal{I}$ and $\mathcal{G}$, respectively) is the problem whether

$$\mathcal{K}_A \cup \mathcal{K}_C \cup \mathcal{E}_{AC1} \cup \mathcal{E}_{\approx} \models (\exists P)\, causes(\mathcal{I}^{-I}, P, \mathcal{G}^{-I})$$

holds

SLDE-Resolution

- Let
  - $\mathcal{K}$ be a set of definite clauses not containing $\approx$ in their heads
  - $\mathcal{E}$ be an equational system and
  - G a goal clause
- Question Does $\mathcal{K} \cup \mathcal{E} \cup \mathcal{E}_{\approx} \models \neg\forall G$ hold?
- Let C be a new variant $H \leftarrow A_1 \wedge \ldots \wedge A_m$ of a clause in $\mathcal{K} \cup \{X \approx X\}$, G the goal clause $\leftarrow B_1 \wedge \ldots \wedge B_n$, and $\mathrm{UP}_{\mathcal{E}}$ an $\mathcal{E}$-unification procedure. If H and B_i, $i \in [1, n]$, are $\mathcal{E}$-unifiable with $\theta \in \mathrm{UP}_{\mathcal{E}}(H, B_i)$ then

$$\leftarrow (B_1 \wedge \ldots \wedge B_{i-1} \wedge A_1 \wedge \ldots \wedge A_m \wedge B_{i+1} \wedge \ldots \wedge B_n)\theta$$

  is called SLDE-resolvent of C and G
- Theorem 4.10
  - SLDE-resolution is sound if $\mathrm{UP}_{\mathcal{E}}$ is sound
  - SLDE-resolution is complete if $\mathrm{UP}_{\mathcal{E}}$ is complete
  - The selection of the literal B_i is don't care non–deterministic

A Solution to Sussman's Anomaly (1)

(1) $\leftarrow causes(ontable(a) \circ ontable(b) \circ on(c,a) \circ clear(b) \circ clear(c) \circ empty,$
$W,$
$ontable(c) \circ on(b,c) \circ on(a,b) \circ clear(a) \circ empty).$

(2) $\leftarrow action(P_1, V_1, Q_1) \wedge$
$P_1 \circ Z_1 \approx ontable(a) \circ ontable(b) \circ on(c,a) \circ clear(b) \circ clear(c) \circ empty \wedge$
$causes(Z_1 \circ Q_1, W_1, ontable(c) \circ on(b,c) \circ on(a,b) \circ clear(a) \circ empty).$

(3) $\leftarrow clear(V_2) \circ on(V_2, W_2) \circ empty \circ Z_1 \approx$
$ontable(a) \circ ontable(b) \circ on(c,a) \circ clear(b) \circ clear(c) \circ empty \wedge$
$causes(Z_1 \circ holding(V_2) \circ clear(W_2),$
$W_1,$
$ontable(c) \circ on(b,c) \circ on(a,b) \circ clear(a) \circ empty).$

(4) $\leftarrow causes(ontable(a) \circ ontable(b) \circ clear(b) \circ clear(a) \circ holding(c),$
$W_1,$
$ontable(c) \circ on(b,c) \circ on(a,b) \circ clear(a) \circ empty).$

$\vdots$

A Solution to Sussman's Anomaly (2)

$$
\begin{array}{ll}
(7) & \leftarrow causes(ontable(a) \circ ontable(b) \circ clear(b) \circ \\
& \qquad clear(a) \circ clear(c) \circ ontable(c) \circ empty,\ W_4, \\
& \qquad ontable(c) \circ on(b,c) \circ on(a,b) \circ clear(a) \circ empty). \\
& \vdots \\
(10) & \leftarrow causes(ontable(a) \circ clear(c) \circ ontable(c) \circ clear(a) \circ holding(b),\ W_7, \\
& \qquad ontable(c) \circ on(b,c) \circ on(a,b) \circ clear(a) \circ empty). \\
& \vdots \\
(14) & \leftarrow causes(ontable(a) \circ ontable(c) \circ clear(a) \circ on(b,c) \circ clear(b) \circ empty,\ W_{10}, \\
& \qquad ontable(c) \circ on(b,c) \circ on(a,b) \circ clear(a) \circ empty). \\
& \vdots \\
(16) & \leftarrow causes(ontable(c) \circ on(b,c) \circ clear(b) \circ holding(a),\ W_{13}, \\
& \qquad ontable(c) \circ on(b,c) \circ on(a,b) \circ clear(a) \circ empty). \\
& \vdots \\
(19) & \leftarrow causes(ontable(c) \circ on(b,c) \circ clear(a) \circ on(a,b) \circ empty,\ W_{16}, \\
& \qquad ontable(c) \circ on(b,c) \circ on(a,b) \circ clear(a) \circ empty). \\
(20) & [\,]
\end{array}
$$

Solving the Frame Problem

- In the fluent calculus the frame problem is mapped onto fluent matching and fluent unification problems
- For example, let

$$s = ontable(a) \circ ontable(b) \circ on(c, a) \circ clear(b) \circ clear(c) \circ empty$$

$$t = clear(c) \circ on(c, a) \circ empty$$

then

$$\theta = \{Z \mapsto ontable(a) \circ ontable(b) \circ clear(b)\}$$

is a most general $\mathcal{E}$–matcher for the $\mathcal{E}$–matching problem

$$\mathcal{E}_{AC1} \cup \mathcal{E}_{\approx} \models (\exists Z)\, s \approx t \circ Z$$

- Consequently, $unstack(c, a)$ can be applied to s yielding

$$s' = ontable(a) \circ ontable(b) \circ clear(b) \circ clear(a) \circ holding(c)$$

Why are States not Modelled by Sets?

- Let $\mathcal{E}_{ACI1} = \mathcal{E}_{AC1} \cup \{X \circ X \approx X\}$
- In this case the $\mathcal{E}$–matching problem

$$\mathcal{E}_{ACI1} \cup \mathcal{E}_{\approx} \models (\exists Z)\, s \approx t \circ Z$$

  has an additional solution, viz.

$$\eta = \{Z \mapsto ontable(a) \circ ontable(b) \circ clear(b) \circ empty\}$$

  θ and η are independent wrt $\mathcal{E}_{ACI1}$
- Computing the successor state in this case yields

$$s'' = ontable(a) \circ ontable(b) \circ clear(b) \circ clear(a) \circ holding(c) \circ empty$$

  which is not intended because the arm of the robot cannot be empty and holding an object at the same time

Remarks (1)

- Some people even believed that the frame problem cannot be solved within first order logic
- Forward versus backward planning
- Many extensions
  - Incomplete specifications of initial situation, e.g.

$$
\begin{array}{l}
(\exists X, P, Y) \\
causes(ontable(b) \circ Y, \\
\qquad\qquad P, \\
\qquad\qquad ontable(c) \circ on(b, c) \circ on(a, b) \circ clear(a) \circ empty \circ X)
\end{array}
$$

  - Indeterminate effects
  - Specificity
  - Ramification and qualification problem

Remarks (2)

- Fluent calculus versus linear logic versus linear connection method
- Fluent calculus versus situation calculus versus event calculus
- Planning problems can be reduced to SAT-problems
  if the length of a plan is restricted