Maximally Permissive Coordination Supervisory Control – Towards Necessary and Sufficient Conditions

Jan Komenda, Tomáš Masopust, and Jan H. van Schuppen

Abstract— **In this paper, we further develop the coordination control framework for discrete-event systems with both complete and partial observation. A new weaker sufficient condition for the computation of the supremal conditionally controllable sublanguage is presented. This result is then used for the computation of the supremal conditionally controllable and conditionally normal sublanguage. The paper further generalizes the previous study by considering general, non-prefix-closed languages.**

I. Introduction

Large scale discrete-event systems (DES) are often formed in a compositional way as a synchronous or asynchronous composition of smaller components, typically automata (or 1-safe Petri nets that can be viewed as products of automata). Supervisory control theory was proposed in [10] for automata as a formal approach that aims to solve the safety issue and nonblockingness.

A major issue is the computational complexity of the centralized supervisory control design, because the global system has an exponential number of states in the number of components. Therefore, a modular supervisory control of DES based on a compositional (local) control synthesis has been introduced and developed by many authors. Structural conditions have been derived for the local control synthesis to equal the global control synthesis in the case of both local and global specification languages.

Specifications are mostly defined over the global alphabet, which means that the global specifications are more relevant than the local specifications. However, several restrictive conditions have to be imposed on the modular plant such as mutual controllability (and normality) of local plant languages for maximal permissiveness of modular control, and other conditions are required for nonblockingness.

For that reason, a coordination control approach was proposed for modular DES in [8] and further developed in [6]. Coordination control can be seen as a reasonable trade-off between a purely modular control synthesis, which is in some cases unrealistic, and a global control synthesis, which is naturally prohibitive for high complexity reasons. The concept of a coordinator is useful for both safety and nonblockingness. The complete supervisor then consists of the coordinator, its supervisor, and the local supervisors for the subsystems. In [8], necessary and sufficient conditions are formulated for nonblockingness and safety, and a sufficient condition is formulated for the maximally permissive control synthesis satisfying a global specification using a coordinator. Later, in [6], a procedure for a distributive computation of the supremal conditionally controllable sublanguage of a given specification has been proposed. We have extended coordination control for non-prefix-closed specification languages in [7] and for partial observations in [4].

In this paper, we first propose a new sufficient condition for a distributive computation of the supremal conditionally controllable sublanguages. We show that it generalizes (is weaker than) both conditions we have introduced earlier in [7] and [6]. Then we revise (simplify) the concepts of conditional observability and conditional normality and present new sufficient conditions for a distributive computation of the supremal conditionally controllable and conditionally normal sublanguage.

The paper is organized as follows. The next section recalls the basic concepts from the algebraic language theory that are needed in this paper. Our coordination control framework is briefly recalled in Section III. In Section IV, new results in coordination control with complete observations are presented: a new, weaker, sufficient condition for distributed computation of supremal conditionally controllable sublanguages. Section V is dedicated to coordination control with partial observations, where the main concepts are simplified. Concluding remarks are in Section VI.

II. Preliminaries

We now briefly recall the elements of supervisory control theory. The reader is referred to [1] for more details. Let Σ be a finite nonempty set of *events*, and let Σ^* denote the set of all finite words (strings) over Σ. The *empty word* is denoted by ε. Let $|\Sigma|$ denote the cardinality of Σ.

A *generator* is a quintuple $G = (Q, \Sigma, f, q_0, Q_m)$, where Q is a finite nonempty set of *states*, Σ is an *event set*, $f : Q \times \Sigma \to Q$ is a *partial transition function*, $q_0 \in Q$ is the *initial state*, and $Q_m \subseteq Q$ is the set of *marked states*. In the usual way, the transition function f can be extended to the domain $Q \times \Sigma^*$ by induction. The behavior of G is described in terms of languages. The language *generated* by G is the set $L(G) = \{s \in \Sigma^* \mid f(q_0, s) \in Q\}$ and the language *marked* by G is the set $L_m(G) = \{s \in \Sigma^* \mid f(q_0, s) \in Q_m\} \subseteq L(G)$.

A *(regular) language* L over an event set Σ is a set $L \subseteq \Sigma^*$ such that there exists a generator G with $L_m(G) = L$. The

J. Komenda and T. Masopust are affiliated with the Institute of Mathematics, Academy of Sciences of the Czech Republic, Žižkova 22, 616 62 Brno, Czech Republic. T. Masopust is also affiliated with the Faculty of Computer Science, TU Dresden, Germany. komenda@math.cas.cz, masopust@math.cas.cz

J. H. van Schuppen is affiliated with Van Schuppen Control Research, Gouden Leeuw 143, 1103 KB, Amsterdam, The Netherlands. jan.h.van.schuppen@xs4all.nl

prefix closure of L is the set $\overline{L} = \{w \in \Sigma^* \mid \text{there exists } u \in \Sigma^* \text{ such that } wu \in L\}$; L is *prefix-closed* if $L = \overline{L}$.

A *(natural) projection* $P : \Sigma^* \to \Sigma_o^*$, for some $\Sigma_o \subseteq \Sigma$, is a homomorphism defined so that $P(a) = \varepsilon$, for $a \in \Sigma \setminus \Sigma_o$, and $P(a) = a$, for $a \in \Sigma_o$. The *inverse image* of P, denoted by $P^{-1} : \Sigma_o^* \to 2^{\Sigma^*}$, is defined as $P^{-1}(s) = \{w \in \Sigma^* \mid P(w) = s\}$. The definitions can naturally be extended to languages. The projection of a generator G is a generator $P(G)$ whose behavior satisfies $L(P(G)) = P(L(G))$ and $L_m(P(G)) = P(L_m(G))$.

A *controlled generator* is a structure (G, Σ_c, P, Γ), where G is a generator over Σ, $\Sigma_c \subseteq \Sigma$ is the set of *controllable events*, $\Sigma_u = \Sigma \setminus \Sigma_c$ is the set of *uncontrollable events*, $P : \Sigma^* \to \Sigma_o^*$ is the projection, and $\Gamma = \{\gamma \subseteq \Sigma \mid \Sigma_u \subseteq \gamma\}$ is the *set of control patterns*. A *supervisor* for the controlled generator (G, Σ_c, P, Γ) is a map $S : P(L(G)) \to \Gamma$. A *closed-loop system* associated with the controlled generator (G, Σ_c, P, Γ) and the supervisor S is defined as the smallest language $L(S/G) \subseteq \Sigma^*$ such that (i) $\varepsilon \in L(S/G)$ and (ii) if $s \in L(S/G)$, $sa \in L(G)$, and $a \in S(P(s))$, then $sa \in L(S/G)$. The marked behavior of the closed-loop system is defined as $L_m(S/G) = L(S/G) \cap L_m(G)$.

Let G be a generator over Σ, and let $K \subseteq L_m(G)$ be a specification. The aim of supervisory control theory is to find a nonblocking supervisor S such that $L_m(S/G) = K$. The nonblockingness means that $\overline{L_m(S/G)} = L(S/G)$, hence $L(S/G) = \overline{K}$. It is known that such a supervisor exists if and only if K is (i) *controllable* with respect to $L(G)$ and Σ_u, that is $\overline{K}\Sigma_u \cap L \subseteq \overline{K}$, (ii) $L_m(G)$*-closed*, that is $K = \overline{K} \cap L_m(G)$, and (iii) *observable* with respect to $L(G)$, Σ_o, and Σ_c, that is for all $s \in \overline{K}$ and $\sigma \in \Sigma_c$, $(s\sigma \notin \overline{K})$ and $(s\sigma \in L(G))$ imply that $P^{-1}[P(s)]\sigma \cap \overline{K} = \emptyset$, where $P : \Sigma^* \to \Sigma_o^*$, cf. [1].

The synchronous product (parallel composition) of languages $L_1 \subseteq \Sigma_1^*$ and $L_2 \subseteq \Sigma_2^*$ is defined by $L_1 \| L_2 = P_1^{-1}(L_1) \cap P_2^{-1}(L_2) \subseteq \Sigma^*$, where $P_i : \Sigma^* \to \Sigma_i^*$, for $i = 1, 2$, are projections to local event sets. In terms of generators, see [1] for more details, it is known that $L(G_1 \| G_2) = L(G_1) \| L(G_2)$ and $L_m(G_1 \| G_2) = L_m(G_1) \| L_m(G_2)$.

III. Coordination Control Framework

A language $K \subseteq (\Sigma_1 \cup \Sigma_2)^*$ is *conditionally decomposable* with respect to event sets Σ_1, Σ_2, and Σ_k, where $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma_k$, if $K = P_{1+k}(K) \parallel P_{2+k}(K)$, where $P_{i+k} : (\Sigma_1 \cup \Sigma_2)^* \to (\Sigma_i \cup \Sigma_k)^*$ is a projection, for $i = 1, 2$. Note that Σ_k can always be extended so that the language K becomes conditionally decomposable. A polynomial algorithm how to compute an extension can be found in [5]. However, to find the minimal extension is NP-hard [7].

Now we recall the coordination control problem that is further developed in this paper.

Problem 1: Consider generators G_1 and G_2 over Σ_1 and Σ_2, respectively, and a generator G_k (called a *coordinator*) over Σ_k with $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma_k$. Assume that a specification $K \subseteq L_m(G_1 \| G_2 \| G_k)$ and its prefix-closure $\overline{K}$ are conditionally decomposable with respect to event sets Σ_1, Σ_2, and Σ_k. The aim of coordination control is to determine nonblocking supervisors S_1, S_2, and S_k for respective generators such that

$$L_m(S_k/G_k) \subseteq P_k(K) \quad \& \quad L_m(S_i/[G_i \| (S_k/G_k)]) \subseteq P_{i+k}(K),$$

for $i = 1, 2$, and

$$L_m(S_1/[G_1 \parallel (S_k/G_k)]) \parallel L_m(S_2/[G_2 \parallel (S_k/G_k)]) = K. \quad \diamond$$

Recall that one way how to construct a coordinator is to set $G_k = P_k(G_1) \parallel P_k(G_2)$, cf. [6], [7].

IV. Coordination Control with Complete Observations

Conditional controllability introduced in [8] and further studied in [3], [4], [6], [7] plays the central role in coordination control. In what follows, we use the notation $\Sigma_{i,u} = \Sigma_i \cap \Sigma_u$ to denote the set of uncontrollable events of the event set Σ_i.

Definition 2 (Conditional controllability): Let G_1 and G_2 be generators over Σ_1 and Σ_2, respectively, and let G_k be a coordinator over Σ_k. A language $K \subseteq L_m(G_1 \| G_2 \| G_k)$ is *conditionally controllable* with respect to generators G_1, G_2, G_k and uncontrollable event sets $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$ if

1) $P_k(K)$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$,
2) $P_{1+k}(K)$ is controllable with respect to $L(G_1) \parallel \overline{P_k(K)}$ and $\Sigma_{1+k,u}$,
3) $P_{2+k}(K)$ is controllable with respect to $L(G_2) \parallel \overline{P_k(K)}$ and $\Sigma_{2+k,u}$,

where $\Sigma_{i+k,u} = (\Sigma_i \cup \Sigma_k) \cap \Sigma_u$, for $i = 1, 2$. $\triangleleft$

The supremal conditionally controllable sublanguage always exists and equals to the union of all conditionally controllable sublanguages [7]. Let

$$\sup \mathrm{cC}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}))$$

denote the supremal conditionally controllable sublanguage of K with respect to $L = L(G_1 \| G_2 \| G_k)$ and sets of uncontrollable events $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$.

The problem is now reduced to determining how to calculate the supremal conditionally-controllable sublanguage.

Consider the setting of Problem 1 and define the languages

$$\boxed{\begin{aligned} \sup \mathrm{C}_k &= \sup \mathrm{C}(P_k(K), L(G_k), \Sigma_{k,u}) \\ \sup \mathrm{C}_{1+k} &= \sup \mathrm{C}(P_{1+k}(K), L(G_1) \parallel \overline{\sup \mathrm{C}_k}, \Sigma_{1+k,u}) \\ \sup \mathrm{C}_{2+k} &= \sup \mathrm{C}(P_{2+k}(K), L(G_2) \parallel \overline{\sup \mathrm{C}_k}, \Sigma_{2+k,u}) \end{aligned}} \qquad (1)$$

where $\sup \mathrm{C}(K, L, \Sigma_u)$ denotes the supremal controllable sublanguage of K with respect to L and Σ_u, see [1] for more details and algorithms.

We have shown that $P_k(\sup \mathrm{C}_{i+k}) \subseteq \sup \mathrm{C}_k$ always holds, for $i = 1, 2$, and that if the converse inclusion holds, we can compute the supremal conditionally-controllable sublanguage in a distributed way.

Theorem 3 ([7]): Consider the setting of Problem 1 and languages defined in (1). If $\sup \mathrm{C}_k \subseteq P_k(\sup \mathrm{C}_{i+k})$, for $i = 1, 2$, then

$$\sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k} = \sup \mathrm{cC}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u})),$$

where $L = L(G_1 \| G_2 \| G_k)$. ■

We can now further improve this result by introducing a weaker condition for nonconflicting supervisors. Recall that two languages L_1 and L_2 are *nonconflicting* if $\overline{L_1 \| L_2} = \overline{L_1} \| \overline{L_2}$.

Theorem 4: Consider the setting of Problem 1 and languages defined in (1). Assume that $\sup \mathrm{C}_{1+k}$ and $\sup \mathrm{C}_{2+k}$ are nonconflicting. If $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k})$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$, then

$$\sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k} = \sup \mathrm{cC}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}))\,,$$

where $L = L(G_1 \| G_2 \| G_k)$.

Proof: Let $\sup \mathrm{cC} = \sup \mathrm{cC}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}))$ and $M = \sup \mathrm{C}_{1+k} \parallel \sup \mathrm{C}_{2+k}$. To prove $M \subseteq \sup \mathrm{cC}$, we show that $M \subseteq P_{1+k}(K) \parallel P_{2+k}(K) = K$ (by conditional decomposability) is conditionally controllable with respect to G_1, G_2, G_k and $\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}$. However, $P_k(M) = P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k})$ (by Lemma 23) is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$ by the assumption. Furthermore, $P_{1+k}(M) = \sup \mathrm{C}_{1+k} \parallel P_k^{2+k}(\sup \mathrm{C}_{2+k})$ implies that $\sup \mathrm{C}_{1+k} \parallel P_k^{1+k}(\sup \mathrm{C}_{1+k}) \parallel P_k^{2+k}(\sup \mathrm{C}_{2+k}) = \sup \mathrm{C}_{1+k} \parallel P_k^{2+k}(\sup \mathrm{C}_{2+k}) = P_{1+k}(M)$. Thus, $P_{1+k}(M) = \sup \mathrm{C}_{1+k} \parallel [P_k^{1+k}(\sup \mathrm{C}_{1+k}) \parallel P_k^{2+k}(\sup \mathrm{C}_{2+k})]$ is controllable with respect to $[L(G_1) \| \overline{\sup \mathrm{C}_k}] \parallel \overline{P_k(M)} = L(G_1) \parallel \overline{P_k(M)}$ by Lemma 21 (because nonconflictingness of $\sup \mathrm{C}_{1+k}$ and $\sup \mathrm{C}_{2+k}$ implies nonconflictingness of $\sup \mathrm{C}_{1+k}$ and $P_k^{1+k}(\sup \mathrm{C}_{1+k}) \parallel P_k^{2+k}(\sup \mathrm{C}_{2+k})$) and by the fact that $P_k^{i+k}(\sup \mathrm{C}_{i+k}) \subseteq \sup \mathrm{C}_k$, for $i = 1, 2$, cf. [7]. Similarly for $P_{2+k}(M)$, hence $M \subseteq \sup \mathrm{cC}$.

To prove the opposite inclusion, it is sufficient to show by Lemma 24 that $P_{i+k}(\sup \mathrm{cC}) \subseteq \sup \mathrm{C}_{i+k}$, for $i = 1, 2$. To prove this note that $P_{1+k}(\sup \mathrm{cC})$ is controllable with respect to $L(G_1) \parallel \overline{P_k(\sup \mathrm{cC})}$ and $\Sigma_{1+k,u}$, and $L(G_1) \parallel \overline{P_k(\sup \mathrm{cC})}$ is controllable with respect to $L(G_1) \parallel \overline{\sup \mathrm{C}_k}$ and $\Sigma_{1+k,u}$ (by Lemma 21) because $P_k(\sup \mathrm{cC})$ being controllable with respect to $L(G_k)$ is also controllable with respect to $\overline{\sup \mathrm{C}_k} \subseteq L(G_k)$. By the transitivity of controllability (Lemma 22), $P_{1+k}(\sup \mathrm{cC})$ is controllable with respect to $L(G_1) \parallel \overline{\sup \mathrm{C}_k}$ and $\Sigma_{1+k,u}$, which implies that $P_{1+k}(\sup \mathrm{cC}) \subseteq \sup \mathrm{C}_{1+k}$. The other case is analogous, hence $\sup \mathrm{cC} \subseteq M$ and the proof is complete. ■

Note that the controllability condition of Theorem 4 is weaker than to require that $\sup \mathrm{C}_k \subseteq P_k(\sup \mathrm{C}_{i+k})$, for $i = 1, 2$.

Proposition 5: If $\sup \mathrm{C}_k \subseteq P_k(\sup \mathrm{C}_{i+k})$, for $i = 1, 2$, then $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k})$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$.

Proof: This is obvious, because due to the converse inclusion being always true we have that $P_k(\sup \mathrm{C}_{i+k}) = \sup \mathrm{C}_k$, for $i = 1, 2$. Hence, $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k}) = \sup \mathrm{C}_k$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$ by definition of $\sup \mathrm{C}_k$. ■

Using the example from [7] we can now show that there are languages such that $\sup \mathrm{C}_k \not\subseteq P_k(\sup \mathrm{C}_{i+k})$, but such that $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k})$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$.

Example 6: Let G_1 and G_2 be generators as shown in Fig. 1, and let K be the language of the generator shown in Fig. 2. Let $\Sigma_c = \{a_1, a_2, c\}$ and $\Sigma_k = \{a_1, a_2, c, u\}$. Let the coordinator $G_k = P_k(G_1) \parallel P_k(G_2)$. Then K is conditionally decomposable, $\sup \mathrm{C}_k = \overline{\{a_1a_2, a_2a_1\}}$, $\sup \mathrm{C}_{1+k} = \overline{\{a_2a_1u_1\}}$, $\sup \mathrm{C}_{2+k} = \overline{\{a_1a_2u_2\}}$, and $\sup \mathrm{C}_k \not\subseteq P_k(\sup \mathrm{C}_{i+k})$. However, $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k}) = \{\varepsilon\}$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$. ◁

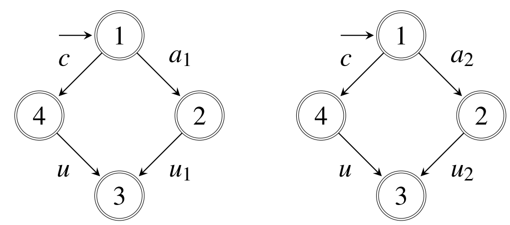

Fig. 1. Generators G_1 and G_2.

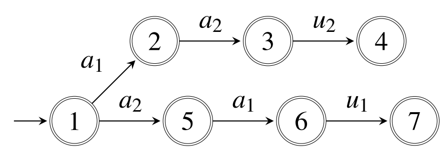

Fig. 2. Specification K.

On the other hand, $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k})$ is not always controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$.

Example 7: Let G_1 and G_2 be generators as shown in Fig. 3, and let K be the language of the generator shown in Fig. 4. Let $\Sigma_c = \{a, c_1, c_2\}$ and $\Sigma_k = \{a, b\}$. Let the coordinator $G_k = P_k(G_1) \parallel P_k(G_2)$. Then the language K is conditionally decomposable, $\sup \mathrm{C}_k = \overline{\{b\}}$, $\sup \mathrm{C}_{1+k} = \overline{\{c_1 b\}}$, $\sup \mathrm{C}_{2+k} = \{\varepsilon\}$, and $P_k(\sup \mathrm{C}_{1+k}) \cap P_k(\sup \mathrm{C}_{2+k}) = \{\varepsilon\}$ is not controllable with respect to $L(G_k) = \overline{\{ab, b\}}$ and $\Sigma_{k,u} = \{b\}$. ◁

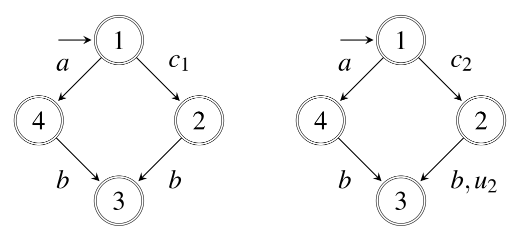

Fig. 3. Generators G_1 and G_2.

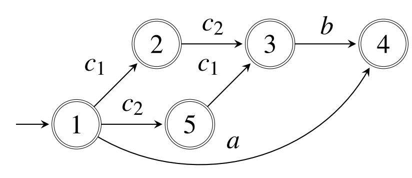

Fig. 4. Specification K.

Recall that it is still an open problem how to compute the supremal conditionally-controllable sublanguage for a general, non-prefix-closed language.

The following conditions were required in [6] to prove the main result for prefix-closed languages. We recall the result here and show that the previous condition is a weaker condition than the one required in [6].

The projection $P:\Sigma^* \to \Sigma_0^*$, where $\Sigma_0 \subseteq \Sigma$, is an *L-observer* for $L \subseteq \Sigma^*$ if, for all $t \in P(L)$ and $s \in \overline{L}$, $P(s)$ is a prefix of t implies that there exists $u \in \Sigma^*$ such that $su \in L$ and $P(su) = t$.

The projection $P:\Sigma^* \to \Sigma_0^*$ is *output control consistent* (OCC) for $L \subseteq \Sigma^*$ if for every $s \in \overline{L}$ of the form $s = \sigma_1 \ldots \sigma_\ell$ or $s = s'\sigma_0\sigma_1 \ldots \sigma_\ell$, $\ell \geq 1$, where $s' \in \Sigma^*$, $\sigma_0, \sigma_\ell \in \Sigma_k$, and $\sigma_i \in \Sigma \setminus \Sigma_k$, for $i = 1,2,\ldots,\ell-1$, if $\sigma_\ell \in \Sigma_u$, then $\sigma_i \in \Sigma_u$, for all $i = 1,2,\ldots,\ell-1$.

The OCC condition can be replaced by a weaker condition called local control consistency (LCC) discussed in [12], [11], see [7]. Let L be a prefix-closed language over Σ, and let Σ_0 be a subset of Σ. The projection $P:\Sigma^* \to \Sigma_0^*$ is *locally control consistent* (LCC) with respect to a word $s \in L$ if for all events $\sigma_u \in \Sigma_0 \cap \Sigma_u$ such that $P(s)\sigma_u \in P(L)$, it holds that either there does not exist any word $u \in (\Sigma \setminus \Sigma_0)^*$ such that $su\sigma_u \in L$, or there exists a word $u \in (\Sigma_u \setminus \Sigma_0)^*$ such that $su\sigma_u \in L$. The projection P is LCC with respect to L if P is LCC for all words of L.

Theorem 8 ([7]): Consider the setting of Problem 1 with a prefix-closed specification K. Consider the languages defined in (1) and assume that $\sup C_{1+k}$ and $\sup C_{2+k}$ are nonconflicting. Let P_k^{i+k} be an $(P_i^{i+k})^{-1}L(G_i)$-observer and OCC (resp. LCC) for $(P_i^{i+k})^{-1}L(G_i)$, for $i = 1,2$. Then

$$\sup C_{1+k} \parallel \sup C_{2+k} = \sup cC(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}))\,,$$

where $L = L(G_1 \| G_2 \| G_k)$. ■

We can now prove that the assumptions of the previous theorem are stronger than the assumptions of Theorem 4. This is shown in the following lemma and corollary, and summarized in Theorem 11.

Lemma 9: Consider the setting of Problem 1 and the languages defined in (1). Assume that $\sup C_{1+k}$ and $\sup C_{2+k}$ are nonconflicting, and let the projection $P_k^{i+k}:(\Sigma_i \cup \Sigma_k)^* \to \Sigma_k^*$ be an $(P_i^{i+k})^{-1}L(G_i)$-observer and OCC (resp. LCC) for $(P_i^{i+k})^{-1}L(G_i)$, for $i = 1,2$. Then $P_k^{1+k}(\sup C_{1+k}) \cap P_k^{2+k}(\sup C_{2+k})$ is controllable with respect to $P_k(L(G_1)) \parallel P_k(L(G_2)) \parallel L(G_k)$ and $\Sigma_{k,u}$.

Proof: Since $\Sigma_{1+k} \cap \Sigma_{2+k} = \Sigma_k$, Lemma 23 implies that $P_k^{1+k}(\sup C_{1+k}) \cap P_k^{2+k}(\sup C_{2+k}) = P_k(\sup C_{1+k} \parallel \sup C_{2+k})$. By Lemma 25, because $P_k^k = id$ is an $L(G_k)$-observer, P_k is an $\overline{L} := \overline{L(G_1 \| G_2 \| G_k)}$-observer. Assume that $t \in \overline{P_k(\sup C_{1+k} \parallel \sup C_{2+k})}$, $u \in \Sigma_{k,u}$, and $tu \in \overline{P_k(L)} = \overline{P_k(L(G_1)) \parallel P_k(L(G_2)) \parallel L(G_k)}$. Then there exists $s \in \overline{\sup C_{1+k} \parallel \sup C_{2+k}} \subseteq \overline{L}$ such that $P_k(s) = t$. By the observer property, there exists v such that $sv \in \overline{L}$ and $P_k(sv) = tu$, that is, $v = v_1u$ with $P_k(v_1u) = u$. By the OCC property, $v_1 \in \Sigma_u^*$, and by controllability of $\sup C_{i+k}$, $i = 1,2$, $sv_1u \in \overline{\sup C_{1+k}} \parallel \overline{\sup C_{2+k}} = \overline{\sup C_{1+k} \parallel \sup C_{2+k}}$, hence $tu \in \overline{P_k(\sup C_{1+k} \parallel \sup C_{2+k})}$.

Similarly for LCC: from $sv = sv_1u \in \overline{L}$, by the LCC property, there exists $v_2 \in (\Sigma_u \setminus \Sigma_k)^*$ such that $sv_2u \in \overline{L}$, and by controllability of $\sup C_{i+k}$, $i = 1,2$, $sv_2u \in \overline{\sup C_{1+k}} \parallel \overline{\sup C_{2+k}} = \overline{\sup C_{1+k} \parallel \sup C_{2+k}}$, hence $tu \in \overline{P_k(\sup C_{1+k} \parallel \sup C_{2+k})}$. ■

Note that if $L(G_k) \subseteq P_k(L(G_1)) \parallel P_k(L(G_2))$, which is actually the way we usually define the coordinator (since we usually define $G_k = P_k(G_1) \parallel P_k(G_2)$), we get the following corollary.

Corollary 10: Consider the setting of Problem 1 with $L(G_k) \subseteq P_k(L(G_1)) \parallel P_k(L(G_2))$ and the languages defined in (1). Assume that $\sup C_{1+k}$ and $\sup C_{2+k}$ are nonconflicting. Let $P_k^{i+k}:(\Sigma_i \cup \Sigma_k)^* \to \Sigma_k^*$ be an $(P_i^{i+k})^{-1}L(G_i)$-observer and OCC (resp. LCC) for $(P_i^{i+k})^{-1}L(G_i)$, for $i = 1,2$. Then $P_k^{1+k}(\sup C_{1+k}) \cap P_k^{2+k}(\sup C_{2+k})$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$.

Proof: The assumption $L(G_k) \subseteq P_k(L(G_1)) \parallel P_k(L(G_2))$ implies that $P_k(L(G_1)) \parallel P_k(L(G_2)) \parallel L(G_k) = L(G_k)$. ■

Finally, as a consequence of Lemma 9 and Theorem 4, we obtain the following result.

Theorem 11: Consider the setting of Problem 1 with $L(G_k) \subseteq P_k(L(G_1)) \parallel P_k(L(G_2))$ and the languages defined in (1). Assume that $\sup C_{1+k}$ and $\sup C_{2+k}$ are nonconflicting. Let P_k^{i+k} be an $(P_i^{i+k})^{-1}L(G_i)$-observer and OCC (resp. LCC) for $(P_i^{i+k})^{-1}L(G_i)$, for $i = 1,2$. Then

$$\sup C_{1+k} \parallel \sup C_{2+k} = \sup cC(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}))\,,$$

where $L = L(G_1 \| G_2 \| G_k)$. ■

V. Coordination Control with Partial Observations

In this section, we study coordination control of modular DES, where both the coordinator supervisor and the local supervisors have incomplete (partial) information about occurrences of their events and, hence, they do not know the exact state of the coordinator and the local plants.

The contribution of this section is twofold. First, basic concepts of conditional observability and conditional normality are simplified in a similar way as it has been done in [7]. Then, we propose new sufficient conditions for a distributed computation of the supremal conditionally normal and conditionally controllable sublanguage. In particular, a weaker condition is presented that combines the weaker condition for distributed computation of the supremal conditionally controllable sublanguage presented in Section IV with a similar condition for computation of the supremal conditionally normal sublanguage. Furthermore, a stronger condition is presented that is easy to check and that works also for non-prefix-closed specifications.

A. Conditional Observability

For coordination control with partial observations, the notion of conditional observability is of the same importance as observability for monolithic supervisory control theory with partial observations.

Definition 12: Let G_1 and G_2 be generators over Σ_1 and Σ_2, respectively, and let G_k be a coordinator over Σ_k. A language $K \subseteq L_m(G_1\|G_2\|G_k)$ is *conditionally observable* with respect to generators G_1, G_2, G_k, controllable sets $\Sigma_{1,c}$, $\Sigma_{2,c}$, $\Sigma_{k,c}$, and projections Q_{1+k}, Q_{2+k}, Q_k, where $Q_i : \Sigma_i^* \to \Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$, if

1) $P_k(K)$ is observable with respect to $L(G_k)$, $\Sigma_{k,c}$, Q_k,
2) $P_{1+k}(K)$ is observable with respect to $L(G_1) \parallel \overline{P_k(K)}$, $\Sigma_{1+k,c}$, Q_{1+k},
3) $P_{2+k}(K)$ is observable with respect to $L(G_2) \parallel \overline{P_k(K)}$, $\Sigma_{2+k,c}$, Q_{2+k},

where $\Sigma_{i+k,c} = \Sigma_c \cap (\Sigma_i \cup \Sigma_k)$, for $i = 1,2$. ◁

Analogously to the notion of $L_m(G)$-closed languages, we recall the notion of conditionally-closed languages defined in [3]. A nonempty language K over Σ is *conditionally closed* with respect to generators G_1, G_2, G_k if

1) $P_k(K)$ is $L_m(G_k)$-closed,
2) $P_{1+k}(K)$ is $L_m(G_1) \parallel P_k(K)$-closed,
3) $P_{2+k}(K)$ is $L_m(G_2) \parallel P_k(K)$-closed.

We can now formulate the main result for coordination control with partial observation. This is a generalization of a similar result for prefix-closed languages given in [4] stated moreover with the above defined simplified (but equivalent) form of conditional observability.

Theorem 13: Consider the setting of Problem 1. There exist nonblocking supervisors S_1, S_2, S_k such that

$$L_m(S_1/[G_1\|(S_k/G_k)]) \parallel L_m(S_2/[G_2\|(S_k/G_k)]) = K \qquad (1)$$

if and only if K is (i) conditionally controllable with respect generators G_1, G_2, G_k and $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$, (ii) conditionally closed with respect to generators G_1, G_2, G_k, and (iii) conditionally observable with respect to G_1, G_2, G_k, event sets $\Sigma_{1,c}$, $\Sigma_{2,c}$, $\Sigma_{k,c}$, and projections Q_{1+k}, Q_{2+k}, Q_k from Σ_i^* to $\Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$.

Proof: (If) Since $K \subseteq L_m(G_1\|G_2\|G_k)$, we have $P_k(K) \subseteq L_m(G_k)$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$, $L_m(G_k)$-closed, and observable with respect to $L(G_k)$, $\Sigma_{k,c}$, and Q_k. It follows, see [1], that there exists a nonblocking supervisor S_k such that $L_m(S_k/G_k) = P_k(K)$. Similarly, we have $P_{1+k}(K) \subseteq L_m(G_1) \parallel L_m(G_k)$ and $P_{1+k}(K) \subseteq (P_k^{1+k})^{-1}P_k(K)$, hence $P_{1+k}(K) \subseteq L_m(G_1) \parallel L_m(G_k) \parallel P_k(K) = L_m(G_1) \parallel P_k(K) = L_m(G_1) \parallel L_m(S_k/G_k)$. This, together with the assumption that K is conditionally controllable, conditionally closed, and conditionally observable imply, see [1], that there exists a nonblocking supervisor S_1 such that $L_m(S_1/[G_1\|(S_k/G_k)]) = P_{1+k}(K)$. A similar argument shows that there exists a nonblocking supervisor S_2 such that $L_m(S_2/[G_2\|(S_k/G_k)]) = P_{2+k}(K)$. Since K is conditionally decomposable, $L_m(S_1/[G_1\|(S_k/G_k)]) \parallel L_m(S_2/[G_2\|(S_k/G_k)]) = P_{1+k}(K) \parallel P_{2+k}(K) = K$.

(Only if) To prove this direction, projections P_k, P_{1+k}, P_{2+k} are applied to (1). The closed-loop languages can be written as synchronous products, thus (1) can be written as $K = L_m(S_1) \parallel L_m(G_1) \parallel L_m(S_k) \parallel L_m(G_k) \parallel L_m(S_2) \parallel L_m(G_2) \parallel L_m(S_k) \parallel L_m(G_k)$, which gives $P_k(K) \subseteq L_m(S_k) \parallel L_m(G_k) = L_m(S_k/G_k)$. On the other hand, $L_m(S_k/G_k) \subseteq P_k(K)$, see Problem 1, hence $L_m(S_k/G_k) = P_k(K)$, which means, according to the basic theorem of supervisory control [1], that $P_k(K)$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$, $L_m(G_k)$-closed, and observable with respect to $L(G_k)$, $\Sigma_{k,c}$, and Q_k. Now, the application of P_{1+k} to (1) gives $P_{1+k}(K) \subseteq L_m(S_1/[G_1\|(S_k/G_k)]) \subseteq P_{1+k}(K)$. According to the basic theorem of supervisory control, $P_{1+k}(K)$ is controllable with respect to $L(G_1\|(S_k/G_k))$ and $\Sigma_{1+k,u}$, $L_m(G_1\|(S_k/G_k))$-closed, and observable with respect to $L(G_1\|(S_k/G_k))$, $\Sigma_{1+k,c}$, and Q_{1+k}. Similarly, $P_{2+k}(K)$ is controllable with respect to $L(G_2\|(S_k/G_k))$ and $\Sigma_{2+k,u}$, $L_m(G_2\|(S_k/G_k))$-closed, and observable with respect to $L(G_2\|(S_k/G_k))$, $\Sigma_{2+k,c}$, and Q_{2+k}, which was to be shown. ■

B. Conditional normality

It is well known that supremal observable sublanguages do not exist in general and it is also the case of conditionally observable sublanguages. Therefore, a stronger concept of language normality has been introduced.

Let G be a generator over Σ, and let $P : \Sigma^* \to \Sigma_o^*$ be a projection. A language $K \subseteq L_m(G)$ is *normal* with respect to $L(G)$ and P if $\overline{K} = P^{-1}P(\overline{K}) \cap L(G)$. It is known that normality implies observability [1].

Definition 14: Let G_1 and G_2 be generators over Σ_1 and Σ_2, respectively, and let G_k be a coordinator over Σ_k. A language $K \subseteq L_m(G_1\|G_2\|G_k)$ is *conditionally normal* with respect to generators G_1, G_2, G_k and projections Q_{1+k}, Q_{2+k}, Q_k, where $Q_i : \Sigma_i^* \to \Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$, if

1) $P_k(K)$ is normal with respect to $L(G_k)$ and Q_k,
2) $P_{1+k}(K)$ is normal with respect to $L(G_1) \parallel \overline{P_k(K)}$ and Q_{1+k},
3) $P_{2+k}(K)$ is normal with respect to $L(G_2) \parallel \overline{P_k(K)}$ and Q_{2+k}. ◁

The following result is an immediate application of conditional normality in coordination control.

Theorem 15: Consider the setting of Problem 1. If the specification K is conditionally controllable with respect to G_1, G_2, G_k and $\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}$, conditionally closed with respect to G_1, G_2, G_k, and conditionally normal with respect to G_1, G_2, G_k and projections Q_{1+k}, Q_{2+k}, Q_k from Σ_i^* to $\Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$, then there exist nonblocking supervisors S_1, S_2, S_k such that

$$L_m(S_1/[G_1\|(S_k/G_k)]) \parallel L_m(S_2/[G_2\|(S_k/G_k)]) = K\,.$$

Proof: As normality implies observability, the proof follows immediately from Theorem 13. ■

The following result was proved for prefix-closed languages in [4]. Here we generalize it for not necessarily prefix-closed languages.

Theorem 16: The supremal conditionally normal sublanguage always exists and equals to the union of all conditionally normal sublanguages.

Proof: We show that conditional normality is preserved under union. Let I be an index set, and let K_i be conditionally normal sublanguages of $K \subseteq L_m(G_1\|G_2\|G_k)$ with respect to generators G_1, G_2, G_k and projections Q_{1+k}, Q_{2+k}, Q_k to local observable event sets, for $i \in I$. We prove that $\bigcup_{i\in I} K_i$ is conditionally normal with respect to those generators and natural projections.

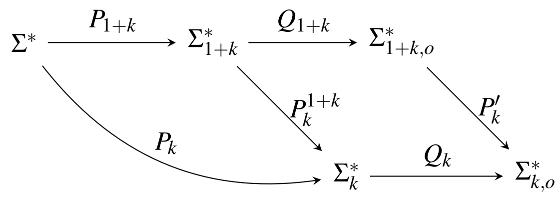

Fig. 5. A commutative diagram of the natural projections.

i) $P_k(\bigcup_{i\in I} K_i)$ is normal with respect to $L(G_k)$ and Q_k because $Q_k^{-1}Q_kP_k(\overline{\bigcup_{i\in I} K_i}) \cap L(G_k) = \bigcup_{i\in I}(Q_k^{-1}Q_kP_k(\overline{K_i}) \cap L(G_k)) = \bigcup_{i\in I} P_k(\overline{K_i}) = P_k(\overline{\bigcup_{i\in I} K_i}) = P_k(\bigcup_{i\in I}\overline{K_i})$, where the second equality is by normality of $P_k(K_i)$ with respect to $L(G_k)$ and Q_k, for $i \in I$.

ii) Note that $Q_{1+k}^{-1}Q_{1+k}P_{1+k}(\overline{\cup_{i\in I} K_i}) \cap L(G_1) \| P_k(\overline{\cup_{i\in I} K_i}) = \cup_{i\in I}(Q_{1+k}^{-1}Q_{1+k}P_{1+k}(\overline{K_i})) \cap \cup_{i\in I}(L(G_1)\|P_k(\overline{K_i})) = \cup_{i\in I}\cup_{j\in I}(Q_{1+k}^{-1}Q_{1+k}P_{1+k}(\overline{K_i}) \cap L(G_1)\|P_k(\overline{K_j}))$ and $P_{1+k}(\overline{\cup_{i\in I} K_i}) \subseteq Q_{1+k}^{-1}Q_{1+k}P_{1+k}(\overline{\cup_{i\in I} K_i}) \cap L(G_1)\|P_k(\overline{\cup_{i\in I} K_i})$. For the sake of contradiction, assume that there exist indexes $i \neq j$ in I such that $Q_{1+k}^{-1}Q_{1+k}P_{1+k}(\overline{K_i}) \cap L(G_1)\|P_k(\overline{K_j}) \not\subseteq P_{1+k}(\overline{\cup_{i\in I} K_i})$. Then the left-hand side must be nonempty, which implies that there exists $x \in Q_{1+k}^{-1}Q_{1+k}P_{1+k}(\overline{K_i}) \cap L(G_1)\|P_k(\overline{K_j})$ and $x \notin P_{1+k}(\overline{\cup_{i\in I} K_i})$. As $x \in Q_{1+k}^{-1}Q_{1+k}P_{1+k}(\overline{K_i})$, there exists $w \in \overline{K_i}$ such that $Q_{1+k}(x) = Q_{1+k}P_{1+k}(w)$. Applying the projection $P'_k : \Sigma^*_{1+k,o} \to \Sigma^*_{k,o}$, we get that $P'_kQ_{1+k}(x) = P'_kQ_{1+k}P_{1+k}(w)$. As $Q_kP_k^{1+k} = P'_kQ_{1+k}$ and $Q_kP_k = P'_kQ_{1+k}P_{1+k}$ (see Fig. 5), we have $Q_kP_k^{1+k}(x) = Q_kP_k(w)$, that is, $P_k^{1+k}(x) \in Q_k^{-1}Q_kP_k(\overline{K_i})$. Since $P_k^{1+k}(x) \in P_k(\overline{K_j}) \subseteq L(G_k)$, the normality of $P_k(K_i)$ with respect to $L(G_k)$ and Q_k gives that $P_k^{1+k}(x) \in P_k(\overline{K_i})$. But then $x \in L(G_1)\|P_k(\overline{K_i})$, and normality of $P_{1+k}(K_i)$ implies that $x \in P_{1+k}(\overline{K_i}) \subseteq P_{1+k}(\overline{\cup_{i\in I} K_i})$, which is a contradiction.

iii) As the last item of the definition is proven in the same way, the theorem holds. ∎

Given generators G_1, G_2, and G_k, let

$$\sup \mathrm{cCN}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}), (Q_{1+k}, Q_{2+k}, Q_k))$$

denote the supremal conditionally controllable and conditionally normal sublanguage of the specification language K with respect to the plant language $L = L(G_1\|G_2\|G_k)$, the sets of uncontrollable events $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$, and projections Q_{1+k}, Q_{2+k}, Q_k, where $Q_i : \Sigma_i^* \to \Sigma_{i,o}^*$, for $i = 1+k, 2+k, k$.

In the sequel, the computation of the supremal conditionally controllable and conditionally normal sublanguage is investigated. In the same way as in [4], the following notation is adopted.

Consider the setting of Problem 1 and define the languages as shown in Fig. 6, where $\sup \mathrm{CN}(K, L, \Sigma_u, Q)$ denotes the supremal controllable and normal sublanguage of K with respect to L, Σ_u, and Q. We recall that the supremal controllable and normal sublanguage always exists and equals the union of all controllable and normal sublanguages of K, cf. [1].

Theorem 17 ([4]): Consider the setting of Problem 1 with a prefix-closed specification K and the languages defined in (2). Let P_k^{i+k} be an $(P_i^{i+k})^{-1}L(G_i)$-observer and OCC (resp. LCC) for $(P_i^{i+k})^{-1}L(G_i)$, for $i = 1, 2$. Assume that the language $P_k^{1+k}(\sup \mathrm{CN}_{1+k}) \cap P_k^{2+k}(\sup \mathrm{CN}_{2+k})$ is normal with respect to $L(G_k)$ and Q_k. Then

$$\begin{aligned}&\sup \mathrm{CN}_{1+k} \parallel \sup \mathrm{CN}_{2+k}\\ &\quad = \sup \mathrm{cCN}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}), (Q_{1+k}, Q_{2+k}, Q_k)),\end{aligned}$$

where $L = L(G_1\|G_2\|G_k)$. ∎

We can now further improve the above result as follows.

Theorem 18: Consider the setting of Problem 1 and the languages defined in (2). Assume that $\sup \mathrm{CN}_{1+k}$ and $\sup \mathrm{CN}_{2+k}$ are nonconflicting and that $P_k^{1+k}(\sup \mathrm{CN}_{1+k}) \cap P_k^{2+k}(\sup \mathrm{CN}_{2+k})$ is controllable and normal with respect to $L(G_k)$, $\Sigma_{k,u}$, and Q_k. Then

$$\begin{aligned}&\sup \mathrm{CN}_{1+k} \parallel \sup \mathrm{CN}_{2+k}\\ &\quad = \sup \mathrm{cCN}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}), (Q_{1+k}, Q_{2+k}, Q_k)),\end{aligned}$$

where $L = L(G_1\|G_2\|G_k)$.

Proof: Let $M = \sup \mathrm{CN}_{1+k} \parallel \sup \mathrm{CN}_{2+k}$ and $\sup \mathrm{cCN} = \sup \mathrm{cCN}(K, L, (E_{1+k,u}, E_{2+k,u}, E_{k,u}), (Q_{1+k}, Q_{2+k}, Q_k))$.

To prove $M \subseteq \sup \mathrm{cCN}$, we show that $M \subseteq P_{1+k}(K) \parallel P_{2+k}(K) = K$ (by conditional decomposability) is conditionally controllable with respect to L and $\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}$ (which follows from Theorem 4), and conditionally normal with respect to L and Q_{1+k}, Q_{2+k}, Q_k (which needs to be shown). However, $P_k(M) = P_k^{1+k}(\sup \mathrm{CN}_{1+k}) \cap P_k^{2+k}(\sup \mathrm{CN}_{2+k})$ is normal with respect to $L(G_k)$ and Q_k by the assumption. Furthermore, $P_{1+k}(M) = \sup \mathrm{CN}_{1+k} \parallel P_k^{2+k}(\sup \mathrm{CN}_{2+k})$. Since $P_{1+k}(M) \subseteq \sup \mathrm{CN}_{1+k}$ and $P_k(M) \subseteq \sup \mathrm{CN}_k$ (by the assumption), $x \in Q_{1+k}^{-1}Q_{1+k}(\overline{P_{1+k}(M)}) \cap L(G_1) \parallel \overline{P_k(M)} \subseteq Q_{1+k}^{-1}Q_{1+k}(\overline{\sup \mathrm{CN}_{1+k}}) \cap L(G_1) \parallel \overline{\sup \mathrm{CN}_k} = \overline{\sup \mathrm{CN}_{1+k}}$ (by normality of $\sup \mathrm{CN}_{1+k}$). In addition, $P_k^{1+k}(x) \in \overline{P_k(M)} \subseteq P_k^{2+k}(\overline{\sup \mathrm{CN}_{2+k}})$. Thus, $x \in \overline{\sup \mathrm{CN}_{1+k}} \parallel P_k^{2+k}(\overline{\sup \mathrm{CN}_{2+k}}) = \overline{P_{1+k}(M)}$ by the nonconflictingness of the supervisors. The case for $P_{2+k}(M)$ is analogous, hence $M \subseteq \sup \mathrm{cCN}$.

To prove $\sup \mathrm{cCN} \subseteq M$, it is sufficient by Lemma 24 to show that $P_{i+k}(\sup \mathrm{cCN}) \subseteq \sup \mathrm{CN}_{i+k}$, for $i = 1, 2$. To do this, note that $P_{1+k}(\sup \mathrm{cCN}) \subseteq P_{1+k}(K)$ is controllable and normal with respect to $L(G_1) \parallel \overline{P_k(\sup \mathrm{cCN})}$, $\Sigma_{1+k,u}$, and Q_{1+k} by definition. Since $P_k(\sup \mathrm{cCN})$ is controllable and normal with respect to $L(G_k)$, $E_{k,u}$, and Q_k, it is also controllable and normal with respect to $\overline{\sup \mathrm{CN}_k} \subseteq L(G_k)$ because $P_k(\sup \mathrm{cCN}) \subseteq \sup \mathrm{CN}_k$. As $P_{1+k}(\sup \mathrm{cCN})$ is controllable with respect to $L(G_1) \parallel \overline{P_k(\sup \mathrm{cCN})}$, and $L(G_1) \parallel \overline{P_k(\sup \mathrm{cCN})}$ is controllable with respect to $L(G_1) \parallel \overline{\sup \mathrm{CN}_k}$ by Lemma 21, the transitivity of controllability (Lemma 22) implies that $P_{1+k}(\sup \mathrm{cCN})$ is controllable with respect to $L(G_1) \parallel \overline{\sup \mathrm{CN}_k}$ and $\Sigma_{1+k,u}$. Similarly, as $P_{1+k}(\sup \mathrm{cCN})$ is normal with respect to $L(G_1) \parallel \overline{P_k(\sup \mathrm{cCN})}$, and $L(G_1) \parallel \overline{P_k(\sup \mathrm{cCN})}$ is normal with respect to $L(G_1) \parallel \overline{\sup \mathrm{CN}_k}$ by Lemma 27, transitivity of normality (Lemma 26) implies that $P_{1+k}(\sup \mathrm{cCN})$ is normal with respect to $L(G_1) \parallel \overline{\sup \mathrm{CN}_k}$ and Q_{1+k}. Thus, we have shown that $P_{1+k}(\sup \mathrm{cCN}) \subseteq \sup \mathrm{CN}_{1+k}$. The case of $P_{2+k}(M)$ is analogous, hence $\sup \mathrm{cCN} \subseteq M$ and the proof is complete. ∎

Note that the sufficient condition in Theorem 18 is not practical for verification, although the intersection is only

$$
\boxed{\begin{aligned}
\sup \mathrm{CN}_k &= \sup \mathrm{CN}(P_k(K), L(G_k), \Sigma_{k,u}, Q_k)\\
\sup \mathrm{CN}_{1+k} &= \sup \mathrm{CN}(P_{1+k}(K), L(G_1) \| \overline{\sup \mathrm{CN}_k}, \Sigma_{1+k,u}, Q_{1+k})\\
\sup \mathrm{CN}_{2+k} &= \sup \mathrm{CN}(P_{2+k}(K), L(G_2) \| \overline{\sup \mathrm{CN}_k}, \Sigma_{2+k,u}, Q_{2+k})
\end{aligned}} \tag{2}
$$

Fig. 6. Definition of supremal controllable and normal sublanguages.

over the coordinator alphabet that is hopefully small. Unlike controllability, normality is not preserved by natural projections under observer and OCC assumptions. This would require results on hierarchical control under partial observations that are not known so far. Therefore, we propose a condition that is (similarly as in the case of complete observations) stronger than the one of Theorem 18, but is easy to check and, moreover, is sufficient for a distributed computation of the supremal conditionally controllable and conditionally normal sublanguage even in the case of non-prefix-closed specifications. Namely, we observe that controllability and normality conditions of Theorem 18 are weaker than to require that $\sup \mathrm{CN}_k \subseteq P_k(\sup \mathrm{CN}_{i+k})$, for $i = 1,2$. The intuition behind the condition $\sup \mathrm{CN}_k \subseteq P_k(\sup \mathrm{CN}_{i+k})$, for $i = 1,2$, is that local supervisors (given by $\sup \mathrm{CN}_{i+k}$) do not need to improve the action by the supervisor for the coordinator on the coordinator alphabet. In this case, the intuition is the same as if the three supervisors (the supervisor for the coordinator and local supervisors) would operate on disjoint alphabets (namely Σ_k, $\Sigma_1 \setminus \Sigma_k$ and $\Sigma_2 \setminus \Sigma_k$) and it is well known that there is no problem with blocking and maximal permissiveness in this case (nonconflictness and mutual controllability of modular control) are trivially satisfied.

Proposition 19: Consider the setting of Problem 1 and the languages defined in (2). If $\sup \mathrm{CN}_k \subseteq P_k(\sup \mathrm{CN}_{i+k})$, for $i = 1,2$, then $P_k(\sup \mathrm{CN}_{1+k}) \cap P_k(\sup \mathrm{CN}_{2+k})$ is controllable and normal with respect to $L(G_k)$, $\Sigma_{k,u}$, and Q_k.

Proof: First of all, we shown that the inclusion $\sup \mathrm{CN}_k \supseteq P_k(\sup \mathrm{CN}_{i+k})$, for $i = 1,2$ always holds true. From its definition, $P_k(\sup \mathrm{CN}_{i+k}) \subseteq P_k(L(G_i) \| \overline{\sup \mathrm{CN}_k}) \subseteq \overline{\sup \mathrm{CN}_k}$ and, clearly, $P_k(\sup \mathrm{CN}_{i+k}) \subseteq P_k(K)$ as well. In order to show that $P_k(\sup \mathrm{CN}_{i+k}) \subseteq \sup \mathrm{CN}_k$, it suffices to show that $\overline{\sup \mathrm{CN}_k} \cap P_k(K) \subseteq \sup \mathrm{CN}_k$. This can be proven by showing that $\overline{\sup \mathrm{CN}_k} \cap P_k(K)$ is controllable and normal with respect to $L(G_k)$, $\Sigma_{k,u}$, and Q_k.

For controllability, let $s \in \overline{\overline{\sup \mathrm{CN}_k} \cap P_k(K)}$, $u \in \Sigma_{k,u}$ with $su \in L(G_k)$. Since there exists $t \in \Sigma_k^*$ such that $st \in \overline{\sup \mathrm{CN}_k} \cap P_k(K) \subseteq \overline{\sup \mathrm{CN}_k}$, we have that $s \in \overline{\sup \mathrm{CN}_k}$ as well. Since $\sup \mathrm{CN}_k$ is controllable with respect to $L(G_k)$ and $\Sigma_{k,u}$, $su \in \overline{\sup \mathrm{CN}_k} \subseteq \overline{P_k(K)}$. Hence, there exists $v \in \Sigma_k^*$ such that $suv \in \sup \mathrm{CN}_k \subseteq P_k(K)$. Altogether, $suv \in \overline{\sup \mathrm{CN}_k} \cap P_k(K)$, i.e., $su \in \overline{\overline{\sup \mathrm{CN}_k} \cap P_k(K)}$.

For normality, let $s \in \overline{\overline{\sup \mathrm{CN}_k} \cap P_k(K)}$ and $s' \in L(G_k)$ with $Q_k(s) = Q_k(s')$. Recall that $s \in \overline{\sup \mathrm{CN}_k}$ as well. Again, normality of $\sup \mathrm{CN}_k$ with respect to $L(G_k)$ and Q_k implies that $s' \in \overline{\sup \mathrm{CN}_k}$. Thus, there exists $v \in \Sigma_k^*$ such that $s'v \in \sup \mathrm{CN}_k \subseteq P_k(K)$. This implies that $s'v \in \overline{\sup \mathrm{CN}_k} \cap P_k(K)$, i.e., $s' \in \overline{\overline{\sup \mathrm{CN}_k} \cap P_k(K)}$, which completes the proof of the inclusion $\sup \mathrm{CN}_k \supseteq P_k(\sup \mathrm{CN}_{i+k})$, for $i = 1,2$.

According to the assumption that the other inclusions also hold, we have the equalities $\sup \mathrm{CN}_k = P_k(\sup \mathrm{CN}_{i+k})$, for $i = 1,2$. Therefore, $P_k(\sup \mathrm{CN}_{1+k}) \cap P_k(\sup \mathrm{CN}_{2+k}) = \sup \mathrm{CN}_k$, which is controllable and normal with respect to $L(G_k)$, $\Sigma_{k,u}$, and Q_k by definition of $\sup \mathrm{CN}_k$. ■

Now, combining Proposition 19 and Theorem 18 we obtain the corollary below.

Corollary 20: Consider the setting of Problem 1 and the languages defined in (2). If $\sup \mathrm{CN}_k \subseteq P_k(\sup \mathrm{CN}_{i+k})$, for $i = 1,2$, then

$$
\begin{aligned}
&\sup \mathrm{CN}_{1+k} \parallel \sup \mathrm{CN}_{2+k}\\
&\quad = \sup \mathrm{cCN}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}), (Q_{1+k}, Q_{2+k}, Q_k)),
\end{aligned}
$$

where $L = L(G_1 \| G_2 \| G_k)$.

Proof: Let $\sup \mathrm{cCN} = \sup \mathrm{cCN}(K, L, (\Sigma_{1,u}, \Sigma_{2,u}, \Sigma_{k,u}), (Q_{1+k}, Q_{2+k}, Q_k))$ and $M = \sup \mathrm{CN}_{1+k} \parallel \sup \mathrm{CN}_{2+k}$. To prove that M is a subset of $\sup \mathrm{cCN}$, we show that (i) M is a subset of K, (ii) M is conditionally controllable with respect to generators G_1, G_2, G_k and uncontrollable event sets $\Sigma_{1,u}$, $\Sigma_{2,u}$, $\Sigma_{k,u}$, and (iii) M is conditionally normal with respect to generators G_1, G_2, G_k and projections Q_{1+k}, Q_{2+k}, Q_k. To this aim, notice that M is a subset of $P_{1+k}(K) \parallel P_{2+k}(K) = K$, because K is conditionally decomposable. Moreover, by Lemma 23 and the fact shown in the proof of Proposition 19 that $\sup \mathrm{CN}_k \supseteq P_k(\sup \mathrm{CN}_{i+k})$, for $i = 1,2$, the language $P_k(M) = P_k(\sup \mathrm{CN}_{1+k}) \cap P_k(\sup \mathrm{CN}_{2+k}) = \sup \mathrm{CN}_k$ is controllable and normal with respect to $L(G_k)$, $\Sigma_{k,u}$, and Q_k. Similarly, $P_{i+k}(M) = \sup \mathrm{CN}_{i+k} \parallel P_k(\sup \mathrm{CN}_{j+k}) = \sup \mathrm{CN}_{i+k} \parallel \sup \mathrm{CN}_k = \sup \mathrm{CN}_{i+k}$, for $j \neq i$, which is controllable and normal with respect to $L(G_i) \parallel \overline{P_k(M)}$. Hence, M is a subset of $\sup \mathrm{cCN}$.

The opposite inclusion is shown in Theorem 18, because nonconflictingness is not needed in this direction of the proof. ■

VI. Conclusion

In this paper, we have further generalized several results of coordination control of concurrent automata with both complete and partial observations. We have presented weaker sufficient conditions for the computation of supremal conditionally controllable sublanguages and supremal conditionally controllable and conditionally normal sublanguages with simplified concepts of conditional observability and conditional normality. Since our results admit quite a straightforward extension to a multi-level coordination control framework, in a future work we would apply our framework to DES models of engineering systems.

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Appendix

In this section, we list the auxiliary results.

Lemma 21 (Proposition 4.6 in [2]): Let $L_i \subseteq \Sigma_i^*$, for $i = 1,2$, be prefix-closed languages, and let $K_i \subseteq L_i$ be controllable with respect to L_i and $\Sigma_{i,u}$. Let $\Sigma = \Sigma_1 \cup \Sigma_2$. If K_1 and K_2 are synchronously nonconflicting, then $K_1 \parallel K_2$ is controllable with respect to $L_1 \parallel L_2$ and Σ_u. ■

Lemma 22 ([6]): Let $K \subseteq L \subseteq M$ be languages over Σ such that K is controllable with respect to $\overline{L}$ and Σ_u, and L is controllable with respect to $\overline{M}$ and Σ_u. Then K is controllable with respect to $\overline{M}$ and Σ_u. ■

Lemma 23 ([13]): Let $P_k : \Sigma^* \to \Sigma_k^*$ be a projection, and let $L_i \subseteq \Sigma_i^*$, where $\Sigma_i \subseteq \Sigma$, for $i = 1,2$, and $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma_k$. Then $P_k(L_1 \parallel L_2) = P_k(L_1) \parallel P_k(L_2)$. ■

Lemma 24 ([6]): Let $L_i \subseteq \Sigma_i^*$, for $i = 1,2$, and let $P_i : (\Sigma_1 \cup \Sigma_2)^* \to \Sigma_i^*$ be a projection. Let $A \subseteq (\Sigma_1 \cup \Sigma_2)^*$ such that $P_1(A) \subseteq L_1$ and $P_2(A) \subseteq L_2$. Then $A \subseteq L_1 \parallel L_2$. ■

Lemma 25 ([9]): Let $L_i \subseteq \Sigma_i^*$, for $i \in J$, be languages, and let $\cup_{k,\ell \in J}^{k \neq \ell}(\Sigma_k \cap \Sigma_\ell) \subseteq \Sigma_0 \subseteq (\cup_{i \in J}\Sigma_i)^*$. If $P_{i,0} : \Sigma_i^* \to (\Sigma_i \cap \Sigma_0)^*$ is an L_i-observer, for $i \in J$, then $P_0 : (\cup_{i \in J}\Sigma_i)^* \to \Sigma_0^*$ is an $(\|_{i \in J} L_i)$-observer. ■

Lemma 26: Let $K \subseteq L \subseteq M$ be languages such that K is normal with respect to L and Q, and L is normal with respect to M and Q. Then, K is normal with respect to M and Q.

Proof: $Q^{-1}Q(\overline{K}) \cap \overline{L} = \overline{K}$ and $Q^{-1}Q(\overline{L}) \cap \overline{M} = \overline{L}$, hence $Q^{-1}Q(\overline{K}) \cap \overline{M} \subseteq Q^{-1}Q(\overline{L}) \cap \overline{M} = \overline{L}$. It implies $Q^{-1}Q(\overline{K}) \cap \overline{M} = Q^{-1}Q(\overline{K}) \cap \overline{M} \cap \overline{L} = \overline{K} \cap \overline{M} = \overline{K}$. ■

Lemma 27: Let $K_1 \subseteq L_1$ over Σ_1 and $K_2 \subseteq L_2$ over Σ_2 be nonconflicting languages such that K_1 is normal with respect to L_1 and $Q_1 : \Sigma_1^* \to \Sigma_{1,o}^*$ and K_2 is normal with respect to L_2 and $Q_2 : \Sigma_2^* \to \Sigma_{2,o}^*$. Then $K_1 \| K_2$ is normal with respect to $L_1 \| L_2$ and $Q : (\Sigma_1 \cup \Sigma_2)^* \to (\Sigma_{1,o} \cup \Sigma_{2,o})^*$.

Proof: $Q^{-1}Q(\overline{K_1 \parallel K_2}) \cap \overline{L_1 \parallel L_2} \subseteq Q_1^{-1}Q_1(\overline{K_1}) \parallel Q_2^{-1}Q_2(\overline{K_2}) \parallel L_1 \parallel L_2 = \overline{K_1} \parallel \overline{K_2} = \overline{K_1 \parallel K_2}$. As the other inclusion always holds, the proof is complete. ■