Exercise Sheet 1: Getting to Know Graphs and the Resource Description Framework

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Exercise 1.1. Show that the number of vertices of odd degree is even in every simple graph.

Exercise 1.2. A *bipartite graph* is a simple graph $G = \langle V, E \rangle$, where V can be partitioned into two sets X, Y (i.e., $X \cup Y = V$, and $X \cap Y = \varnothing$), such that every edge $\{ a, b \} \in E$ coincides with both X and Y, i.e., $\{ a, b \} \cap X \neq \varnothing$ and $\{ a, b \} \cap Y \neq \varnothing$.

Show that the following are equivalent:

1. $G = \langle V, E \rangle$ is bipartite.
2. G is *2-colourable*, i.e., there is a map $c : V \to \{ 0, 1 \}$ such that no two adjacent vertices a, b have the same colour, i.e., $c(a) \neq c(b)$ for all $\{ a, b \} \in E$.
3. G does not contain a cycle $v_1 \xrightarrow{e_1} v_2 \xrightarrow{e_2} \cdots \xrightarrow{e_{n-1}} v_n \xrightarrow{e_n} v_1$ of odd length.

Exercise 1.3. Write a (Python) program that takes as input a directed graph in the format given below, and prints out all vertices that have maximal out-degree. The input should be read from a file given as a command-line argument.

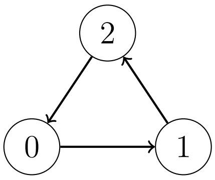
The file format is as follows:

$$\begin{array}{cc} n & \\ s_1 & t_1 \\ s_2 & t_2 \\ s_3 & t_3 \\ \vdots & \vdots \\ s_m & t_m \end{array}$$

The first line consists of a single integer n, the number of vertices of the graph. Each of the following lines consists of two integers s_i and t_i, specifying an edge from vertex s_i to vertex t_i, separated by a space. Vertices are numbered $0, 1, \ldots, n-1$.

As an example, the following input encodes a directed triangle:

```
3
0 1
1 2
2 0
```



Data files are available on Github[1].

[1] https://github.com/knowsys/Course-Knowledge-Graphs/tree/main/data/simple-graphs

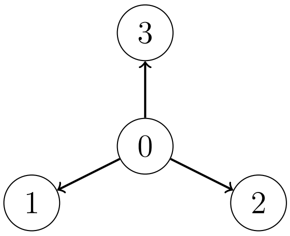
Exercise 1.4. Write a program that reads a directed graph from a file in the format of Exercise 1.3 and prints out the graph in METIS graph format:

$$\begin{array}{lllll}
n & m & & & \\
v_1 & n_1^1 & n_1^2 & \cdots & n_1^{d_1} \\
v_2 & n_2^1 & n_2^2 & \cdots & n_2^{d_2} \\
\vdots & \vdots & \vdots & \cdots & \vdots \\
v_n & n_n^1 & n_n^2 & \cdots & n_n^{d_n}
\end{array}$$

The first line consists of two integers n and m, separated by a space, where n is the number of vertices, and m is the total number of edges. Each of the following lines specifies the neighbours $n_i^1, n_i^2, \ldots, n_i^{d_i}$ of vertex v_i.

As an example, the directed star S_3 would be encoded as:

```
4 3
0 1 2 3
```



Exercise 1.5. A triangle in a directed graph is a simple directed path $v_1 \xrightarrow{e_1} v_2 \xrightarrow{e_2} v_3 \xrightarrow{e_3} v_1$.

Write a program that reads a directed graph G from a file in the format of Exercise 1.3 and prints out the number of triangles in G. How does the runtime of your program scale with the size of the input graph?

Exercise 1.6. Write a program that reads a graph in N-Triples format and checks whether the graph is bipartite. Use it to decide whether `authorship.nt.gz`[2] and `coauthors.nt.gz`[2] are bipartite.

Hint: each of the uncompressed graphs is roughly 4 GiB in size. In Python, you can use `gzip.GzipFile`[3] to process the compressed file without decompressing it first. There is also `authorship-snippet.nt.gz`[2], a small part of the graph that you can use during development.

Please note: In order to get the correct data files, please install git-lfs[4] on your system, and then activate it in your local repository (git lfs install).

[2] https://github.com/knowsys/Course-Knowledge-Graphs/tree/main/data/dblp

[3] https://docs.python.org/3/library/gzip.html

[4] https://git-lfs.github.com/