



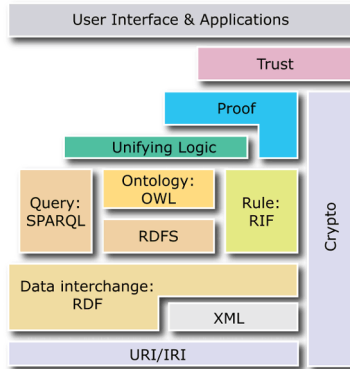
TECHNISCHE
UNIVERSITÄT
DRESDEN

FOUNDATIONS OF SEMANTIC WEB TECHNOLOGIES

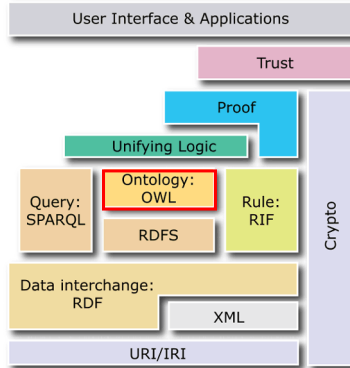
Hypertableau I

Sebastian Rudolph

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Agenda

- Motivation
- Recap: Translation into FOL
- Structural Transformation
- Translation into Clauses
- Summary

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Example Standard Tableau

Let the following TBox \mathcal{T} and ABox \mathcal{A} be given:

$$\mathcal{T} = \{ \exists r.A \sqsubseteq A \} \quad C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$\mathcal{A} = \{ \neg A(a_0), r(a_0, b_1), r(b_1, a_1), \dots, r(a_{n-1}, b_n), r(b_n, a_n), A(a_n) \}$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \cdots \cdots \cdots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

Assumption: we address the nodes in the tableau in alphabetic order, i.e., a's before b's

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$$L(a_{n-1}) = \{C_{\mathcal{T}}\}$$

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Now again decisions for a_n and b_1, \dots, b_n

Is that Necessarily So?

- the algorithm constructs exponentially many branches, despite dependency directed backtracking
- translation of the formula into FOL:

$$\begin{aligned} & \forall r. (\neg A) \sqcup A \\ &= \forall x, y. [r(x, y) \wedge A(y) \rightarrow A(x)] \\ &= \forall x, y. [\neg r(x, y) \vee \neg A(y) \vee A(x)] \end{aligned}$$

- note: the formula does not have real non-determinism (Horn-clause)
- hypertableau exploits this

Idea Hypertableau

- translate KB axioms into FOL
 - rewrite axioms to obtain formulae of a certain structure
- axioms are translated such that non-determinism is avoided, if possible
- the formulae thus obtained become rules for constructing a model abstraction

Simple Hypertableau Example

1 Translation into Clauses

$$A \sqsubseteq A'$$

$$A \sqsubseteq \exists r.B$$

$$D \sqsubseteq E \sqcup F$$

$$F \sqsubseteq \perp$$

$$\exists r.T \sqsubseteq C$$

$$A(a)$$

$$(D \sqcap \neg B)(d)$$

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- existential quantifiers treated as in the tableau

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$$\rightarrow A(a)$$

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Simple Hypertableau Example

1 Translation into Clauses

$A \sqsubseteq A'$	$A(x) \rightarrow A'(x)$
$A \sqsubseteq \exists r.B$	$A(x) \rightarrow \exists r.B(x)$
$D \sqsubseteq E \sqcup F$	$D(x) \rightarrow E(x) \vee F(x)$
$F \sqsubseteq \perp$	$F(x) \rightarrow \perp(x)$
$\exists r.T \sqsubseteq C$	$r(x, y) \rightarrow C(x)$
$A(a)$	$\rightarrow A(a)$
$(D \sqcap \neg B)(d)$	$\rightarrow D(d)$
	$\rightarrow \neg B(d)$

- existential quantifiers treated as in the tableau

Simple Hypertableau Example

 a d

$$L(a) = \{A\}$$

$$L(d) = \{D, \neg B\}$$

$$\begin{aligned} A(x) &\rightarrow A'(x) \\ A(x) &\rightarrow \exists r.B(x) \\ D(x) &\rightarrow E(x) \vee F(x) \\ F(x) &\rightarrow \perp(x) \\ r(x, y) &\rightarrow C(x) \\ &\rightarrow A(a) \\ &\rightarrow D(d) \\ &\rightarrow \neg B(d) \end{aligned}$$

Simple Hypertableau Example

a

d

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Simple Hypertableau Example

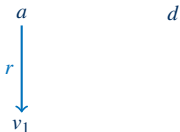
 a d

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Simple Hypertableau Example



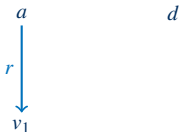
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Simple Hypertableau Example



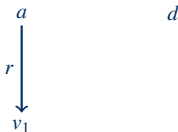
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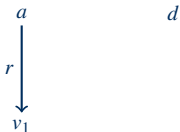
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 \end{array}$$

- no more rules applicable \rightsquigarrow satisfiability has been shown
- the only thing left from the tableau rules: an analogue of the \exists -rule

Agenda

- Motivation
- **Recap: Translation into FOL**
- Structural Transformation
- Translation into Clauses
- Summary

Translation into FOL

translation of TBox axioms into FOL via the mapping π with C, D complex classes, r a role and A an atomic class:

$$\pi(C \sqsubseteq D) = \forall x. (\pi_x(C) \rightarrow \pi_x(D)) \quad \pi(C \equiv D) = \forall x. (\pi_x(C) \leftrightarrow \pi_x(D))$$

$$\pi_x(A) = A(x)$$

$$\pi_y(A) = A(y)$$

$$\pi_x(\neg C) = \neg \pi_x(C)$$

$$\pi_y(\neg C) = \neg \pi_y(C)$$

$$\pi_x(C \sqcap D) = \pi_x(C) \wedge \pi_x(D)$$

$$\pi_y(C \sqcap D) = \pi_y(C) \wedge \pi_y(D)$$

$$\pi_x(C \sqcup D) = \pi_x(C) \vee \pi_x(D)$$

$$\pi_y(C \sqcup D) = \pi_y(C) \vee \pi_y(D)$$

$$\pi_x(\forall r. C) = \forall y. (r(x, y) \rightarrow \pi_y(C))$$

$$\pi_y(\forall r. C) = \forall x. (r(y, x) \rightarrow \pi_x(C))$$

$$\pi_x(\exists r. C) = \exists y. (r(x, y) \wedge \pi_y(C))$$

$$\pi_y(\exists r. C) = \exists x. (r(y, x) \wedge \pi_x(C))$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$\pi(C \sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D)))$$

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$$\pi(C \sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D)))$$
$$\forall x. [\pi_x(C) \rightarrow \pi_x(\exists r. (\forall s. (D \sqcup \exists r. D)))]$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$\begin{aligned} & \pi(C \sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D))) \\ \forall x. & [\pi_x(C) \rightarrow \pi_x(\exists r. (\forall s. (D \sqcup \exists r. D)))] \\ \forall x. & [C(x) \rightarrow \exists y. (r(x, y) \wedge \pi_y(\forall s. (D \sqcup \exists r. D)))] \end{aligned}$$

Motivation Normal Form

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$$\begin{aligned} & \pi(C \sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D))) \\ \forall x. & [\pi_x(C) \rightarrow \pi_x(\exists r. (\forall s. (D \sqcup \exists r. D)))] \\ \forall x. & [C(x) \rightarrow \exists y. (r(x, y) \wedge \pi_y(\forall s. (D \sqcup \exists r. D)))] \\ \forall x. & [C(x) \rightarrow \exists y. (r(x, y) \wedge \forall x. (s(y, x) \rightarrow \pi_x(D \sqcup \exists r. D)))] \\ \forall x. & [C(x) \rightarrow \exists y. (r(x, y) \wedge \forall x. (s(y, x) \rightarrow \pi_x(D) \vee \pi_x(\exists r. D)))] \end{aligned}$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

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Structural Transformation

- Structural Transformation introduces new concepts for complex subconcepts

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Structural Transformation

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$$\begin{aligned} C &\sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D)) \\ \rightsquigarrow C &\sqsubseteq \exists r. Q_1 \\ Q_1 &\equiv \forall s. (D \sqcup \exists r. D) \end{aligned}$$

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Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

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Well. Is that correct? Let $\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$

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- In the context of a TBox with the original axiom, the ABox was contradictory.
- In the context of a TBox with the rewritten axiom, the ABox is satisfiable

Polarity for Optimized Transformation

- we have to take care if subexpressions occur “positively” or “negatively”
- $A \sqsubseteq B$ is just $\neg A \sqcup B$
- thus, A occurs negatively in the axiom and B positively
- when replacing a negatively occurring concept, we have to use \sqsupseteq

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$$C_{\mathcal{T}} = (\forall r.(\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r. (\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

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$$L(a) = \{\neg C\}$$

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Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r. (\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

$$\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$$



$$L(a) = \{\neg C\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q), \neg A\}$$

$$L(b) = \{A, B\} \cup \{\neg Q\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q)\}$$

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no further choice options
 \rightsquigarrow the KB is satisfiable

Optimized Structural Transformation

- we now want to define structural transformation formally
- we want to introduce just one new concept name per subexpression
- goal: rewrite TBox into an equisatisfiable TBox umzuschreiben containing only “simple” axioms

Polarity of Concepts

We define the polarity of a concept C inside a formula as follows:

- C occurs in C positively,
- C occurs in $\neg D$ positively (negatively) if C occurs in D negatively (positively),
- C occurs in $D \sqcap E$ or $D \sqcup E$ positively (negatively), if C occurs positively (negatively) in D or E ,
- C occurs in $\exists r.D$ or $\forall r.D$ positively (negatively), if C occurs positively (negatively) in D ,
- C occurs in $D \sqsubseteq E$ positively (negatively), if C occurs positively (negatively) in E or negatively (positively) in D .

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A concept occurs positively (negatively) in an (\mathcal{ALC}) TBox \mathcal{T} , if C occurs positively (negatively) in an axiom in \mathcal{T} .

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A concept occurs positively (negatively) in an (\mathcal{ALC}) TBox \mathcal{T} , if C occurs positively (negatively) in an axiom in \mathcal{T} .

\rightsquigarrow a concept may occur both positively and negatively in an axiom

Optimized Transformation with Polarity

Let \mathcal{T} be an \mathcal{ALC} TBox. For every concept (sub-)expression C in \mathcal{T} , we introduce a fresh atomic concept A_C and define the function $\text{st}(C)$ as follows:

$$\begin{array}{lll} \text{st}(A) = A & \text{st}(\neg C) = \neg A_C & \text{st}(\exists r.C) = \exists r.A_C \\ \text{st}(\top) = \top & \text{st}(C \sqcap D) = A_C \sqcap A_D & \text{st}(\forall r.C) = \forall r.A_C \\ \text{st}(\perp) = \perp & \text{st}(C \sqcup D) = A_C \sqcup A_D & \end{array}$$

The result of the structural transformation of a TBox \mathcal{T} is a TBox \mathcal{T}' with the following axioms:

- $A_C \sqsubseteq \text{st}(C)$ for every concept C occurring positively in \mathcal{T} ,
- $\text{st}(C) \sqsubseteq A_C$ for every concept C occurring negatively in \mathcal{T} ,
- $A_C \sqsubseteq A_D$ for every GCI $C \sqsubseteq D \in \mathcal{T}$.

Simplification of Axioms

we can now use known equivalences to simplify the axioms further:

$$\{C \sqsubseteq D \sqcap E\} \equiv \{C \sqsubseteq D, C \sqsubseteq E\}$$

$$\{C \sqcup D \sqsubseteq E\} \equiv \{C \sqsubseteq E, D \sqsubseteq E\}$$

Result of the Structural Transformation

by virtue of structural transformation and the known equivalences, we can rewrite an \mathcal{ALC} KB into an equisatisfiable one, which contains only axioms of the following shape (A and B being atomic):

$$A_1 \sqcap \dots \sqcap A_n \sqsubseteq B_1 \sqcup \dots \sqcup B_m$$

$$A \sqsubseteq \exists r.B$$

$$A \sqsubseteq \forall r.B$$

$$\exists r.A \sqsubseteq B$$

$$\forall r.A \sqsubseteq B$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\underbrace{\forall r.(A \sqcap D)}_{C_3} \sqcap A}_{C_1} \sqcap \underbrace{\underbrace{\exists s.(A \sqcap D)}_{C_4}}_{C_1}}^{C_7} \sqsubseteq B \sqcup \underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E) \sqcap F}_{C_5}}_{C_2}}_{C_8} \sqcup \underbrace{\forall r.A}_{C_6}$$

$\underbrace{\hspace{15em}}_{C_9}$

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$$A \sqcap D \sqsubseteq C_1$$

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$$\begin{aligned}
 A \sqcap D &\sqsubseteq C_1 \\
 C_2 &\sqsubseteq C \sqcap E
 \end{aligned}$$

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$$A \sqcap D \sqsubseteq C_1$$

$$C_2 \sqsubseteq C \sqcap E$$

$$\forall r.C_1 \sqsubseteq C_3$$

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 C_5 &\sqsubseteq \exists r.C_3
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Example: Optimized Structural Transformation

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Example: Optimized Structural Transformation

we still can apply the simplification rules:

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$$A \sqcap D \sqsubseteq C_1$$

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$$C_5 \sqsubseteq \exists r.C_3$$

$$C_2 \sqsubseteq E$$

$$C_6 \sqsubseteq \forall r.A$$

$$C_3 \sqcap A \sqcap C_4 \sqsubseteq C_7$$

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$$C_9 \sqsubseteq B \sqcup C_8 \sqcup C_6$$

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Agenda

- Motivation
- Recap: Translation into FOL
- Structural Transformation
- Translation into Clauses
- Summary

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

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$$A_1(x) \wedge \dots \wedge A_n(x) \rightarrow B_1(x) \vee \dots \vee B_m(x)$$

If $m = 0$, the rule head contains $\perp(x)$.

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$$A \sqsubseteq \exists r.B \qquad A(x) \rightarrow (\exists r.B)(x)$$
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If $m = 0$, the rule head contains $\perp(x)$.

$A \sqsubseteq \exists r.B$	$A(x) \rightarrow (\exists r.B)(x)$
$A \sqsubseteq \forall r.B$	$A(x) \wedge r(x, y) \rightarrow B(y)$
$\exists r.A \sqsubseteq B$	$r(x, y) \wedge A(y) \rightarrow B(x)$
$\forall r.A \sqsubseteq B$	$\rightarrow (\exists r. \neg A)(x) \vee B(x)$

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Summary

- axioms can be simplified via structural transformation
- simplified axioms can be expressed as rules
- existential quantification in rule heads allowed
- often these rules allow to avoid nondeterminism