TECHNISCHE UNIVERSITÄT DRESDEN

Artificial Intelligence, Computational Logic

# PROBLEM SOLVING AND SEARCH IN ARTIFICIAL INTELLIGENCE

## Lecture 1

**Sarah Gaggl**

Dresden, 8th April 2016

DRESDEN concept
Exzellenz aus Wissenschaft und Kultur

# What to expect

- The course has 12 lectures, 6 tutorials and a practical part
- Lecture is on Friday in DS 4, 13:00-14:30
- Tutorials are on Friday in DS 5 14:50-16:20 and consist of exercise sheets
- Schedule and lecture material will be available at course web-page `https://ddll.inf.tu-dresden.de/web/Problem_Solving_and_Search_in_Artificial_Intelligence_(SS2016)`
- The practical part consists of solving (implementation) a problem and its presentation. Should be performed in groups of two, assignments will be ready at April 29th.
- 3 fixed Dates for practical part (see web-page)
    1) Analysis of the problem, group building
    2) Concept how to solve it
    3) Presentation of solution - last questions
- DEADLINE?
- EXAM?

Literature


- Zbigniew Michalewicz and David B. Fogel. How to Solve It: Modern Heuristics, volume 2. Springer, 2004.
- Stuart J. Russell and Peter Norvig. Artificial Intelligence - A Modern Approach (3. edition ). Pearson Education, 2010.
- plus additional articles

# Agenda

1. Introduction
2. Uninformed Search versus Informed Search (Best First Search, A* Search, Heuristics)
3. Local Search, Stochastic Hill Climbing, Simulated Annealing
4. Tabu Search
5. Answer-set Programming (ASP)
6. Constraint Satisfaction (CSP)
7. Structural Decomposition Techniques (Tree/Hypertree Decompositions)
8. Evolutionary Algorithms/ Genetic Algorithms

# What are the Ages of my Three Sons?

Two men meet on the street. One gives the other a puzzle

A: "All three of my sons celebrate their birthday this very day! So, can you tell me how old each of them is?"

B: "Sure, but you'll have to tell me something about them."

A: "The product of the ages of my sons is 36."

B: "That's fine but I need more than just this."

A: "The sum of their ages is equal to the number of windows in that building."

B: "Still, I need an additional hint to solve your puzzle."

A: "My oldest son has blue eyes."

B: "Oh, this is sufficient!"

# What are the Ages of my Three Sons? ctd.

"The product of the ages of my sons is 36."

| son 1 | son 2 | son 3 |
|---|---|---|
| 36 | 1 | 1 |
| 18 | 2 | 1 |
| 12 | 3 | 1 |
| 9 | 4 | 1 |
| 9 | 2 | 2 |
| 6 | 6 | 1 |
| 6 | 3 | 2 |
| 4 | 3 | 3 |

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| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 36 | + | 1 | + | 1 | = | 38 |
| 18 | + | 2 | + | 1 | = | 21 |
| 12 | + | 3 | + | 1 | = | 16 |
| 9 | + | 4 | + | 1 | = | 14 |
| 9 | + | 2 | + | 2 | = | 13 |
| 6 | + | 6 | + | 1 | = | 13 |
| 6 | + | 3 | + | 2 | = | 11 |
| 4 | + | 3 | + | 3 | = | 10 |

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## What was difficult on this problem?

# Problem Solving

- Where to begin?
- You have to create the plan for generating a solution.
- Always consider all of the available data.
- Can you make connections between the goal and what is given?

# Why are Some Problems Difficult to Solve?

- The number of possible solutions in the search space is too large for an exhaustive search.
- The problem is too complicated, and simplified models of the problem are useless.
- The evaluation function of the quality of a solution is noisy or varies with time, which requires an entire series of solutions.
- There are so many constraints that finding even one feasible answer is difficult, let alone searching for an optimal solution.
- The person solving the problem is inadequately prepared.

# The Size of the Search Space

## Boolean Satisfiability Problem (SAT)

Make a compound statement of Boolean variables evaluate to TRUE.

- For example, consider the following problem of 100 variables given in conjunctive normal form (CNF):

$$F(x) = (x_{17} \vee \neg x_{37} \vee x_{73}) \wedge (\neg x_{11} \vee \neg x_{56}) \wedge \cdots \wedge (x_2 \vee x_{43} \vee \neg x_{77} \vee \neg x_{89} \vee \neg x_{97}).$$

- Challenge: find the truth assignment for each variable $x_i$, for all $i = 1, \ldots 100$ s.t. $F(x) = \text{TRUE}$.

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Space of possible solutions.

- Any binary string of length 100 is a possible solution.
- Two choices for each variable, and taken over 100 variables, generates $2^{100}$ possibilities.

# The Size of the Search Space ctd.

- Size of the search space $\mathcal{S}$ is $|\mathcal{S}| = 2^{100} \approx 10^{30} = 1\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000.$
- The number of bacterial cells on Earth is estimated at around $5 \times 10^{30}$.
- If we had a computer that could test 1000 strings per second and could have started at the beginning of time itself, 15 billion years ago (Big Bang!) we would have examined fewer than $1\%$ of all the possibilities by now!
- Trying out all alternatives is out of the question.
- Choice of which evaluation function to use.
- Solutions closer to the right answer should yield better evaluations than those who are far away.
- If we try a string $x$ and $F(x)$ returns TRUE, we are done. But what if $F(x)$ returns FALSE?
- How to find a function which gives more than just "right" or "wrong"?

# The Size of the Search Space ctd.

## Traveling Salesperson Problem (TSP)

- Given $n$ cities and the distances between each pair of cities;
- Traveling salesperson must visit every city exactly once and return home covering the shortest distance.

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## Seach Space

- Set of permutations of $n$ cities.
- $2n$ different ways (for symmetrical TSP) to represent one tour.
- There are $n!$ ways to permute $n$ numbers.
- $|\mathcal{S}| = n!/(2n) = (n-1)!/2$

# The Size of the Search Space ctd.

- $|\mathcal{S}| = n!/(2n) = (n-1)!/2$
- For any $n > 6$, number of possible solutions to the TSP with $n$ cities is larger than the number of possible solutions to the SAT problem with $n$ variables.
- For $n = 6$: $5!/2 = 60$ solutions to the TSP and $2^6 = 64$ solutions to a SAT.
- For $n = 7$: 360 solutions to the TSP and 128 to the SAT.
- Search space increases very quickly with increasing $n$.
- A 50-city TSP has more solutions than existing liters of water on the planet.
- However, the evaluation function for the TSP is more straightforward than for SAT.
- Table with distances between each pair of cities.
- After $n$ addition operations we could calculate the distance of any candidate tour and use this to evaluate its merit.
- $cost = dist(15,3) + dist(3,11) + \cdots + dist(6,15)$

# Modeling the problem

- We only find the solution to a model of the problem.
- All models are simplifications of the real world.
- Problem $\rightarrow$ Model $\rightarrow$ Solution
    1. Use an approximate model of a problem and find the precise solution: Problem $\rightarrow$ Model$_a$ $\rightarrow$ Solution$_p$(Model$_a$)
    2. Use a precise model of the problem and find an approximate solution: Problem $\rightarrow$ Model$_p$ $\rightarrow$ Solution$_a$(Model$_p$)
- Which one is better?

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  2. Use a precise model of the problem and find an approximate solution: Problem $\rightarrow$ Model$_p$ $\rightarrow$ Solution$_a$(Model$_p$)
- Which one is better?
- Solution$_a$(Model$_p$) is better than Solution$_p$(Model$_a$).

# Change over time

Problems my change

- before you model them,
- while you derive a solution, and
- after you execute the solution.

TSP - Travel time between two cities depends on many factors:

- traffic lights
- slow-moving trucks
- flat tire
- weather
- many more...

# Constraints

- Almost all practical problems pose constraints
- Two types of constraints:
  - Hard constraints, and
  - Soft constraints.
- Constraints make the search space smaller, but
  - It is hard to create operators that will act on feasible solution and generate in turn new feasible solutions that are an improvement of previous solution.
  - The geometry of search space gets tricky.

# Constraints ctd.

## Timetable of the classes at a college in one semester

We are given

- list of courses that are offered;
- list of students assigned to each class;
- professors assigned to each class;
- list of available classrooms, and information for size and other facilities that each offer.

# Constraints ctd.

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- list of available classrooms, and information for size and other facilities that each offer.

Construct timetables that fulfill hard constraints:

- Each class must be assigned to an available room that has enough seats and requisite facilities.
- Students who are enrolled in more than one class can not have their classes held at the same time on the same day.
- Professors can not be assigned to teach courses that overlap in time.

# Constraints ctd.

## Timetable - Soft Constraints:

- Courses that meets twice a week should preferably be assigned to Mondays and Wednesdays or Tuesdays and Thursdays.
- Courses that meets three times per week should preferably be assigned to Mondays, Wednesdays, and Fridays.
- Course time should be assigned so that students do not have to take final exams for multiple courses without any break in between.
- If more than one room satisfies the requirements for a course and is available at the designated time, the course should be assigned to the room with the capacity that is closest to the class size.

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- Any timetable that meets the hard constraints is feasible.
- The timetable has to be optimized in the light of soft constraints.
- Each soft constraint has to be quantified.
- We can evaluate two candidate assignments and decide that one is better than other.

# Solve the Problem!

- Mr. Smith and his wife invited four other couples for a party.
- When everyone arrived, some of the people in the room shook hands with some of the others.
- Nobody shook hands with their spouse and nobody shook hands with the same person twice.
- After that, Mr. Smith asked everyone how many times they shook someone's hand.
- He received different answers from everybody.
- How many times did Mrs. Smith shake someone's hand?

# Summary

Problem solving is difficult for several reasons:

- Complex problems often pose an enormous number of possible solutions.
- To get any sort of solution at all, we often have to introduce simplifications that make the problem tractable. As a result, the solutions that we generate may not be very valuable.
- The conditions of the problem change over time and might even involve other people who want to fail you.
- Real-world problems often have constraints that require special operations to generate feasible solutions.

# References

Zbigniew Michalewicz and David B. Fogel.
**How to Solve It: Modern Heuristics**, volume 2. Springer, 2004.