Exercise 1

[Hard?] Prove Hanf's locality theorem for FO (see slides/notes from the 6th lecture for a hint).

Exercise 2

Employ tilings to show that the satisfiability problem for the three-variable fragment FO^3 of FO is undecidable.

Exercise 3

Show that the problem of checking if two input FO formulae are equivalent (are satisfies by exactly the same structures) is undecidable.

Exercise 4

Analyse lecture notes and convince yourself that you understand why query evaluation in FO is in PSPACE in combined complexity but only LOGSPACE in data complexity.

Exercise 5

QBF is a well-known PSPACE-complete problem. Provide a polynomial reduction from QBF to the query evaluation problem for FO and conclude its PSPACE-completness. Hint: Two-element structure suffices.

Exercise 6

Show that FO^1 is decidable (you can use only the variable x in formulae). Hint: Prove that if an input φ has a model then it has a "small" model.

Exercise 7

Read slides on MSO games (slides 18–20 from HERE) and prove that even is not $MSO[\emptyset]$ definable.