FAKULTÄT FÜR !NFORMATIK

Faculty of Informatics

dbai

cf2 Semantics Revisited

Sarah Alice Gaggl and Stefan Woltran

Institute of Informationsystems, Vienna University of Technology

Desenzano del Garda — September 8th, 2010

FACULTY OF !NFORMATICS

WWTF

Wiener Wissenschafts-, Forschungs- und Technologiefonds

Motivation

- *cf2* semantics satisfies symmetric treatment of odd- and even-length cycles.
- Need for a uniform platform for comparison of different semantics.
- Many semantics already encoded within Answer-Set Programming (ASP).
- *cf2* semantics is rather cumbersome to be implemented directly in ASP due to the recursive computation of different sub-frameworks.
- We provide an alternative characterization for the verification problem of the *cf2* semantics which enables us to directly
  - guess a set S,
  - check whether S is maximal conflict-free in an instance of the given framework.

Overview

1. Preliminaries
   - Definitions and Notations
   - Original Definition of $cf2$
2. Alternative Characterization
   - Recursively Component Defeated Sets
   - Fixed-point Characterization
   - Main Result
3. ASP-Encodings
4. Conclusion

Definition (Argumentation Framework)

An argumentation framework (AF) is a pair $F = (A, R)$, where A is a finite set of arguments and $R \subseteq A \times A$. Then $(a, b) \in R$ if a attacks b.
Let $cf(F)$ be the collection of conflict-free sets in F, then $S \in cf(F)$ if $\forall a, b \in S$, $(a, b) \notin R$.
Let $mcf(F)$ be the collection of maximal conflict-free sets of F, then $S \in mcf(F)$ if $S \in cf(F)$ and $\forall T \in cf(F)$, $S \not\subset T$.

Further Notations:

- $SCCs(F)$: set of strongly connected components of F,
- $C_F(a)$: the unique set $C \in SCCs(F)$, s. t. $a \in C$,
- $F|_S = ((A \cap S), R \cap (S \times S))$: sub-framework of $F = (A, R)$ wrt a set S,
- $F|_S - S' = F|_{S \setminus S'}$.

Original Definition of *cf2*

Definition ($D_F(S)$)

Let $F = (A, R)$ be an AF and $S \subseteq A$. An argument $b \in A$ is component-defeated by S (in F), if there exists an $a \in S$, such that $(a, b) \in R$ and $a \notin C_F(b)$. The set of arguments component-defeated by S in F is denoted by $D_F(S)$.

Definition (*cf2*)

Let $F = (A, R)$ be an argumentation framework and S a set of arguments. Then, S is a *cf2* extension of F, i.e. $S \in cf2(F)$, iff

- in case $|SCCs(F)| = 1$, then $S \in mcf(F)$,
- otherwise, $\forall C \in SCCs(F)$, $(S \cap C) \in cf2(F|_C - D_F(S))$.

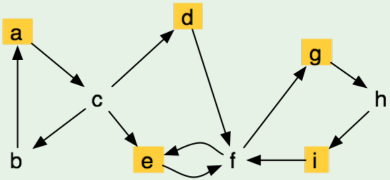
Original Definition of *cf2* (cont.)

Definition (*cf2*)

- in case $|SCCs(F)| = 1$, then $S \in mcf(F)$,
- otherwise, $\forall C \in SCCs(F)$, $(S \cap C) \in cf2(F|_C - D_F(S))$.

Example

$S = \{a, d, e, g, i\}$, $S \in cf2(F)$?



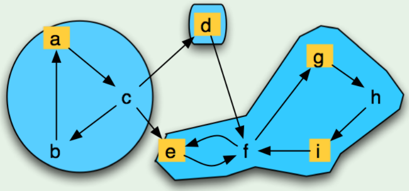
Original Definition of *cf2* (cont.)

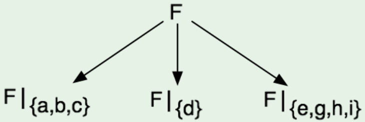
Definition (*cf2*)

- in case $|SCCs(F)| = 1$, then $S \in mcf(F)$,
- otherwise, $\forall C \in SCCs(F)$, $(S \cap C) \in cf2(F|_C - D_F(S))$.

Example

$S = \{a, d, e, g, i\}$, $S \in cf2(F)$? $C_1 = \{a, b, c\}$, $C_2 = \{d\}$, $C_3 = \{e, f, g, h, i\}$ and $D_F(S) = \{f\}$.





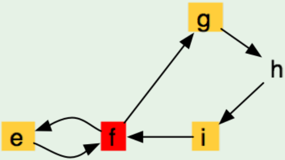
Original Definition of *cf2* (cont.)

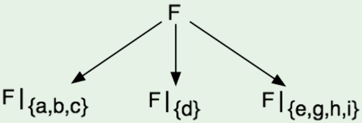
Definition (*cf2*)

- in case $|SCCs(F)| = 1$, then $S \in mcf(F)$,
- otherwise, $\forall C \in SCCs(F)$, $(S \cap C) \in cf2(F|_C - D_F(S))$.

Example

$S = \{a, d, e, g, i\}$, $S \in cf2(F)$? $C_1 = \{a, b, c\}$, $C_2 = \{d\}$, $C_3 = \{e, f, g, h, i\}$ and $D_F(S) = \{f\}$.



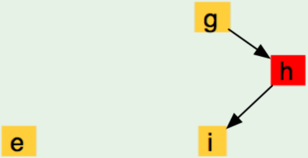


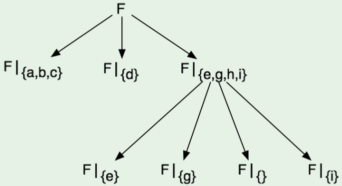
Definition (*cf2*)

- in case $|SCCs(F)| = 1$, then $S \in mcf(F)$,
- otherwise, $\forall C \in SCCs(F)$, $(S \cap C) \in cf2(F|_C - D_F(S))$.

Example

$S = \{a, d, e, g, i\}$, $S \in cf2(F)$? $C_4 = \{e\}$, $C_5 = \{g\}$, $C_6 = \{h\}$, $C_7 = \{i\}$ and $D_{F|_{\{e,g,h,i\}}}(\{e, g, i\}) = \{h\}$.





Definition ($\mathcal{RD}_F(S)$)

Let $F = (A, R)$ be an AF and S a set of arguments. We define the set of arguments recursively component defeated by S (in F) as follows:

- if $|SCCs(F)| = 1$ then $\mathcal{RD}_F(S) = \emptyset$;
- otherwise, $\mathcal{RD}_F(S) = D_F(S) \cup \bigcup_{C \in SCCs(F)} \mathcal{RD}_{F|_{C - D_F(S)}}(S \cap C)$.

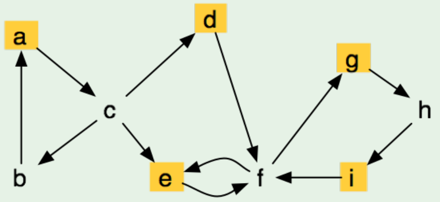
Recursively Component Defeated Sets

Definition ($\mathcal{RD}_F(S)$)

- if $|SCCs(F)| = 1$ then $\mathcal{RD}_F(S) = \emptyset$;
- otherwise, $\mathcal{RD}_F(S) = D_F(S) \cup \bigcup_{C \in SCCs(F)} \mathcal{RD}_{F|_{C - D_F(S)}}(S \cap C)$.

Example

$S = \{a, d, e, g, i\}$



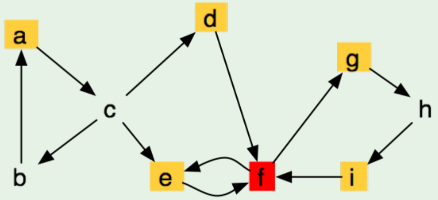
FACULTY OF !NFORMATICS

Definition ($\mathcal{RD}_F(S)$)

- if $|SCCs(F)| = 1$ then $\mathcal{RD}_F(S) = \emptyset$;
- otherwise, $\mathcal{RD}_F(S) = D_F(S) \cup \bigcup_{C \in SCCs(F)} \mathcal{RD}_{F|_{C-D_F(S)}}(S \cap C)$.

Example

$S = \{a, d, e, g, i\}$, $\mathcal{RD}_F(S) = \{f\} \cup \bigcup_{C \in SCCs(F)} \mathcal{RD}_{F|_{C-\{f\}}}(S \cap C)$.



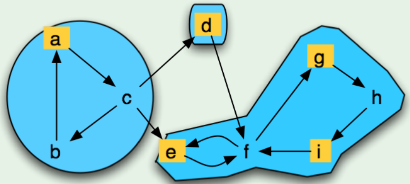
Recursively Component Defeated Sets

Definition ($\mathcal{RD}_F(S)$)

- if $|SCCs(F)| = 1$ then $\mathcal{RD}_F(S) = \emptyset$;
- otherwise, $\mathcal{RD}_F(S) = D_F(S) \cup \bigcup_{C \in SCCs(F)} \mathcal{RD}_{F|_{C - D_F(S)}}(S \cap C)$.

Example

$S = \{a, d, e, g, i\}$, $\mathcal{RD}_F(S) = \{f\} \cup \bigcup_{C \in SCCs(F)} \mathcal{RD}_{F|_{C - \{f\}}}(S \cap C)$.



FACULTY OF !NFORMATICS

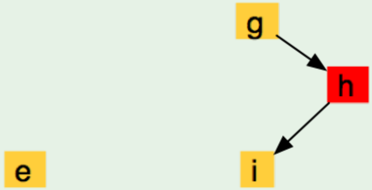
Recursively Component Defeated Sets

Definition ($\mathcal{RD}_F(S)$)

- if $|SCCs(F)| = 1$ then $\mathcal{RD}_F(S) = \emptyset$;
- otherwise, $\mathcal{RD}_F(S) = D_F(S) \cup \bigcup_{C \in SCCs(F)} \mathcal{RD}_{F|_{C - D_F(S)}}(S \cap C)$.

Example

$\mathcal{RD}_{F|_{\{e,g,h,i\}}}(\{e, g, i\}) = \{h\}$.

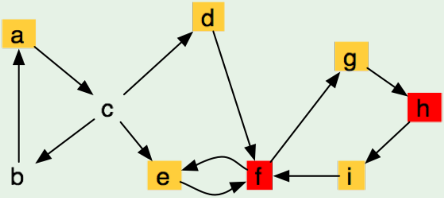


Definition ($\mathcal{RD}_F(S)$)

- if $|SCCs(F)| = 1$ then $\mathcal{RD}_F(S) = \emptyset$;
- otherwise, $\mathcal{RD}_F(S) = D_F(S) \cup \bigcup_{C \in SCCs(F)} \mathcal{RD}_{F|_{C - D_F(S)}}(S \cap C)$.

Example

$S = \{a, d, e, g, i\}$, $\mathcal{RD}_F(S) = \{f, h\}$.

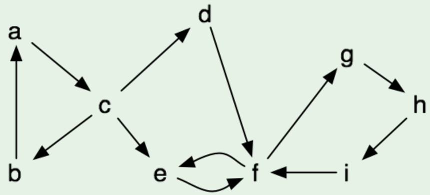


Separation

Definition (Separation)

An AF $F = (A, R)$ is called separated if for each $(a, b) \in R$, $C_F(a) = C_F(b)$. We define $[[F]] = \bigcup_{C \in SCCs(F)} F|_C$ and call $[[F]]$ the separation of F.

Example

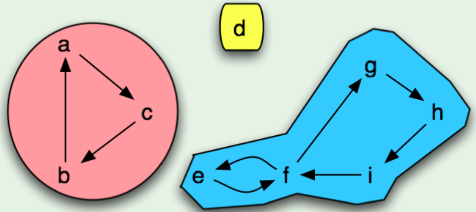


Separation

Definition (Separation)

An AF $F = (A, R)$ is called separated if for each $(a, b) \in R$, $C_F(a) = C_F(b)$. We define $[[F]] = \bigcup_{C \in SCCs(F)} F|_C$ and call $[[F]]$ the separation of F.

Example



Alternative Characterization

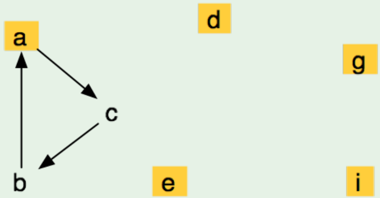
Lemma (1)

Let $F = (A, R)$ be an AF and S be a set of arguments. Then,

$$S \in cf2(F) \text{ iff } S \in mcf([[F - \mathcal{RD}_F(S)]]).$$

Example

$S = \{a, d, e, g, i\}$, $\mathcal{RD}_F(S) = \{f, h\}$, $S \in mcf([[F - \mathcal{RD}_F(S)]])$.



Definition (Reachability)

Let $F = (A, R)$ be an AF, B a set of arguments, and $a, b \in A$. We say that b is reachable in F from a modulo B, in symbols $a \Rightarrow_F^B b$, if there exists a path from a to b in $F|_B$.

Fixed-point Characterization

Definition (Reachability)

Let $F = (A, R)$ be an AF, B a set of arguments, and $a, b \in A$. We say that b is reachable in F from a modulo B, in symbols $a \Rightarrow_F^B b$, if there exists a path from a to b in $F|_B$.

Definition ($\Delta_{F,S}$)

For an AF $F = (A, R)$, $D \subseteq A$, and a set S of arguments,

$$\Delta_{F,S}(D) = \{a \in A \mid \exists b \in S : b \neq a, (b, a) \in R, a \not\Rightarrow_F^{A \setminus D} b\}.$$

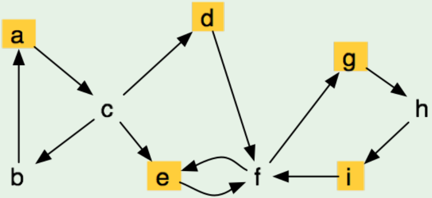
FACULTY OF !NFORMATICS

$\Delta_{F,S}$

$$\Delta_{F,S}(D) = \{a \in A \mid \exists b \in S : b \neq a, (b,a) \in R, a \not\Rightarrow_F^{A \setminus D} b\}.$$

Example

$S = \{a, d, e, g, i\}$.

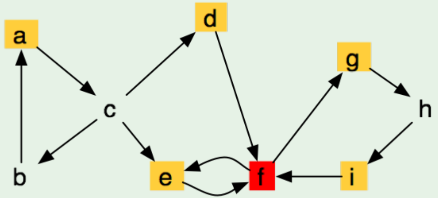


$\Delta_{F,S}$

$$\Delta_{F,S}(\emptyset) = \{a \in A \mid \exists b \in S : b \neq a, (b,a) \in R, a \not\Rightarrow^{A}_{F} b\}.$$

Example

$S = \{a, d, e, g, i\}$, $\Delta_{F,S}(\emptyset) = \{f\}$.



Example (cont.)

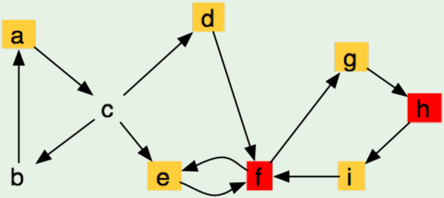
$\Delta_{F,S}$

$$\Delta_{F,S}(D) = \{a \in A \mid \exists b \in S : b \neq a, (b,a) \in R, a \not\Rightarrow_F^{A \setminus D} b\}.$$

Example

$S = \{a, d, e, g, i\}$,

$\Delta_{F,S}(\{f\}) = \{a \in A \mid \exists b \in S : b \neq a, (b,a) \in R, a \not\Rightarrow_F^{A \setminus \{f\}} b\} = \{f, h\}$.



Lemma (2)

For any AF $F = (A, R)$ *and any set* $S \in cf(F)$, $\Delta_{F,S} = \mathcal{RD}_F(S)$.

Main Result

Lemma (2)

For any AF $F = (A, R)$ *and any set* $S \in cf(F)$, $\Delta_{F,S} = \mathcal{RD}_F(S)$.

Theorem

For any AF F, $cf2(F) = \{S \mid S \in cf(F) \cap mcf([[F - \Delta_{F,S}]])\}$.

Main Result

Lemma (2)

For any AF $F = (A, R)$ *and any set* $S \in cf(F)$, $\Delta_{F,S} = \mathcal{RD}_F(S)$.

Theorem

For any AF F, $cf2(F) = \{S \mid S \in cf(F) \cap mcf([[F - \Delta_{F,S}]])\}$.

Proof sketch.

- Lemma (1): $S \in cf2(F)$ iff $S \in mcf([[F - \mathcal{RD}_F(S)]])$,
- Lemma (2) and
- $S \in cf2(F)$ implies $S \in cf(F)$ [Baroni et al.].

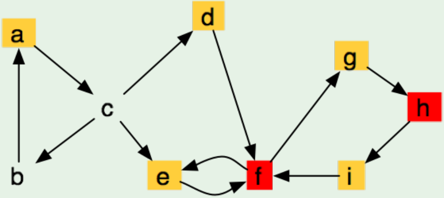
□

Theorem

For any AF F, $cf2(F) = \{S \mid S \in cf(F) \cap mcf([[F - \Delta_{F,S}]])\}$.

Example

$S \in cf(F)$, $\Delta_{F,S} = \{f, h\}$.



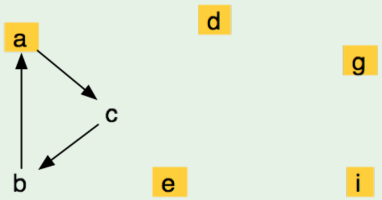
Example (cont.)

Theorem

For any AF F, $cf2(F) = \{S \mid S \in cf(F) \cap mcf([[F - \Delta_{F,S}]])\}$.

Example

$\Delta_{F,S} = \{f, h\}$, $[[F - \Delta_{F,S}]] = \big(\{a, b, c, d, e, g, i\}, \{(a, b), (b, c), (c, a)\}\big)$,
$S \in mcf([[F - \Delta_{F,S}]])$.

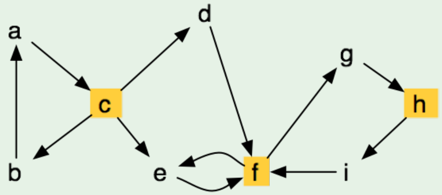


Theorem

For any AF F, $cf2(F) = \{S \mid S \in cf(F) \cap mcf([[F - \Delta_{F,S}]])\}$.

Example

$S = \{c, f, h\}$, $S \in cf(F)$.



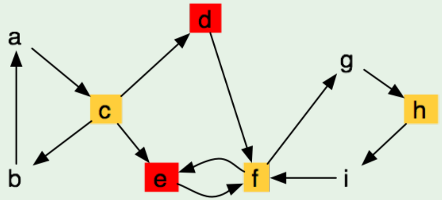
Example (cont.)

Theorem

For any AF F, $cf2(F) = \{S \mid S \in cf(F) \cap mcf([[F - \Delta_{F,S}]])\}$.

Example

$S = \{c, f, h\}$, $\Delta_{F,S}(\emptyset) = \{d, e\}$.



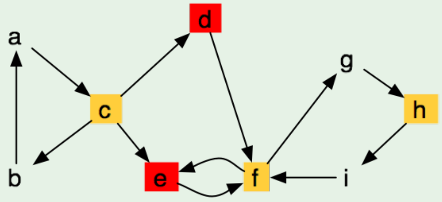
Example (cont.)

Theorem

For any AF F, $cf2(F) = \{S \mid S \in cf(F) \cap mcf([[F - \Delta_{F,S}]])\}$.

Example

$S = \{c, f, h\}$, $\Delta_{F,S}(\{d,e\}) = \{d,e\}$.



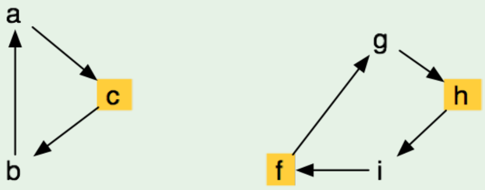
Example (cont.)

Theorem

For any AF F, $cf2(F) = \{S \mid S \in cf(F) \cap mcf([[F - \Delta_{F,S}]])\}$.

Example

$S = \{c, f, h\}$, $\Delta_{F,S} = \{d, e\}$, $S \in mcf([[F - \Delta_{F,S}]])$.



Given an AF $F = (A, R)$, we identify the following Guess & Check procedure:

1. Guess the conflict-free sets $S \subseteq A$ of F.
2. For each S, compute the set $\Delta_{F,S}$.
3. For each S, derive the instance $[[F - \Delta_{F,S}]]$.
4. Check whether S is maximal conflict-free in $[[F - \Delta_{F,S}]]$.

ASP-Encodings

Given an AF $F = (A, R)$, we identify the following Guess & Check procedure:

1. Guess the conflict-free sets $S \subseteq A$ of F.
2. For each S, compute the set $\Delta_{F,S}$.
3. For each S, derive the instance $[[F - \Delta_{F,S}]]$.
4. Check whether S is maximal conflict-free in $[[F - \Delta_{F,S}]]$.

The encodings for the *cf2* semantics are incorporated in the system

ASPARTIX

`http://rull.dbai.tuwien.ac.at:8080/ASPARTIX`

Conclusion

- Alternative characterization for the *cf2* semantics.
- By shifting the recursion into the fixed-point operator $\Delta_{F,S}$, we avoid the recursive generation of sub-frameworks.
- This allows for a succinct ASP-encoding which has also been incorporated into ASPARTIX.

Conclusion

- Alternative characterization for the *cf2* semantics.
- By shifting the recursion into the fixed-point operator $\Delta_{F,S}$, we avoid the recursive generation of sub-frameworks.
- This allows for a succinct ASP-encoding which has also been incorporated into ASPARTIX.

Software Demo

We will present our web application of ASPARTIX at the Software Demo Session on Thursday, 9th September.

Thank you!