





Finite-Controllability of Conjunctive Queries in the \mathcal{Z} family of Description Logics (Extended Abstract)*

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The description logics (DLs) from the \mathcal{Z} family [3] are among the most powerful KR formalisms on the verge of decidability [7,2]. For brevity we assume that TBoxes of \mathcal{ZIQ} , \mathcal{ZOI} and \mathcal{ZOQ} are in a normal form. A normal form TBox can be computed from an original TBox in polynomial time and is a conjunction of concept definitions of the following form:

$$A \equiv \{o\}, A \equiv \neg B, A \equiv B \sqcup B', A \equiv \geq ns.B, A \equiv \exists p_{\mathbb{A}}.B, A \equiv \exists s.\text{Self}$$

where A, B, B' are concept names, s is a simple role, o is an individual name and $p_{\mathbb{A}}$ is a so-called automaton role with \mathbb{A} being an NFA. *Simple roles* s are generated by the following grammar: $s ::= r \mid s \cap s \mid s \cup s \mid s \setminus s$, where r is an atomic role or the inverse of an atomic role. Automaton roles are interpreted in a natural way, e.g. $d \in (\exists p_{\mathbb{A}}.B)^{\mathcal{I}}$, if there is an element in $B^{\mathcal{I}}$ and a path leading to it from $d \in \Delta^{\mathcal{I}}$ that is accepted by \mathbb{A} . In \mathcal{ZIQ} , \mathcal{ZOQ} we exclude, respectively, nominals and inverse roles, and in \mathcal{ZOI} we assume that $n = 1$.

Important computational problems, like the knowledge-base satisfiability problem (KBSAT) and the query entailment problem (QEnt), for $\mathcal{ZIQ}/\mathcal{ZOQ}/\mathcal{ZOI}$ are well understood. They are EXPTIME-complete [4] and 2EXPTIME-complete, respectively, even for the class of positive regular two-way queries [4,2]. Surprisingly, in stark contrast with those results, nearly nothing is known about the behaviour of the \mathcal{Z} family of DLs in the finite-world scenario. The only result that we are aware of is a quite recent proof of the finite model property of \mathcal{ZOI} from [4]. Thus, starting a quest for understanding the finite model reasoning in the \mathcal{Z} family, we investigate the decidability/complexity of KBSAT and QEnt for the class of local queries, i.e. for conjunctive queries (CQs) and unions thereof (UCQs).

Let \mathcal{K} be a $\mathcal{ZOI}/\mathcal{ZOQ}$ knowledge base and let q be a UCQ. If there exists an interpretation \mathcal{I} satisfying \mathcal{K} but not q , we say that \mathcal{I} is a countermodel for \mathcal{K} and q . A description logic is called *finitely-UCQ-controllable* if for all its knowledge bases and UCQs, if there is a countermodel for them there is also a finite one. Our main technical result is:

Theorem 1. *\mathcal{ZOI} and \mathcal{ZOQ} are finitely-UCQ-controllable.*

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This yields also the finite model property of $\mathcal{Z}\mathcal{O}\mathcal{Q}$ and $\mathcal{Z}\mathcal{O}\mathcal{I}$, thus complementing the results of [4]. Harvesting from the complexity results of [3,2] we conclude:

Corollary 2. *Finite knowledge-base satisfiability problem for $\mathcal{Z}\mathcal{O}\mathcal{I}$ and $\mathcal{Z}\mathcal{O}\mathcal{Q}$ is EXPTIME-complete, while their finite CQ/UCQ query entailment is 2EXPTIME-complete.*

Roughly speaking, our proof method is as follows. We start from an infinite forest-shaped finite branching countermodel \mathcal{I} for a $\mathcal{Z}\mathcal{O}\mathcal{I}/\mathcal{Z}\mathcal{O}\mathcal{Q}$ knowledge base and a query q (with K atoms), on which we can additionally assume that whether an element requires a witness for some existential restriction, then such a witness is always reachable by a path going downward the forest. Next, we identify in \mathcal{I} the set of all possible *downward types* π , *i.e.* the isomorphism types of subtrees of depth K rooted at some element from \mathcal{I} . For every downward type π we select one representative, extend it by providing witnesses for $\exists p_{\mathbb{A}}.B$ requirements of its root (note that a path accepted by \mathbb{A} is finite by definition) and make such a substructure sibling- and parent closed. Its domain is denoted with Δ_{π} . Such modified representatives are called components: they serve as the main building blocks in the construction of a finite countermodel for \mathcal{K} and q . For technical reasons we distinguish nominal types and components, *i.e.* the ones whose roots are roots of \mathcal{I} .

Let L be the maximal number of leaves across all the components and let M be the maximal number of children of a node in \mathcal{I} . The domain of our finite countermodel \mathcal{J} is:

$$\Delta^{\mathcal{J}} := \bigcup_{\text{nominal components } \pi} \Delta_{\pi} \quad \cup \quad \bigcup_{\text{non-nominal components } \pi} \Delta_{\pi, \ell, m}^{\pi', b},$$

where decorated Δ_{π} are just disjoint copies of Δ_{π} , π' ranges over nominal types, $1 \leq \ell \leq L$, $1 \leq m \leq M$ and $b \in \{0, 1\}$. For every π, π', ℓ, m, b as above, we make $\mathcal{D}_{\pi, \ell, m}^{\pi', b} := \mathcal{J} \upharpoonright \Delta_{\pi, \ell, m}^{\pi', b}$ isomorphic to the corresponding component from \mathcal{I} and similarly $\mathcal{D}_{\pi} := \mathcal{J} \upharpoonright \Delta_{\pi}$ for nominal components. What remains to be done is to define roles between different components of \mathcal{J} . For every component, its leaves are connected to the roots of other components, in order to provide the leaves their local witnesses. For every π the root of $\mathcal{D}_{\pi, \ell, m}^{\pi', 1-b}$ will serve as the m -th child of the ℓ -th leaf of any of the components $\mathcal{D}_{\pi', *, *}^{*, b}$, or, in the case when $b = 1$ and π' is nominal, of the component $\mathcal{D}_{\pi'}$. This finishes the construction of \mathcal{J} .

It can be shown that \mathcal{J} indeed preserves the given knowledge base satisfiability and the query. For the former task it is very convenient that we use automaton roles instead of regular expressions which are used in the original definition of the logics from \mathcal{Z} . For the latter it is important that our building blocks are of depth at least K .

For the logic $\mathcal{Z}\mathcal{I}\mathcal{Q}$, for which the finite and unrestricted KBSAT do not coincide (see e.g. [1, Theorem 3.19] for a much weaker logic), we provide an exponential time reduction from finite conjunctive query entailment to its finite KBSAT. This is optimal, modulo the conjecture that finite KBSAT for $\mathcal{Z}\mathcal{I}\mathcal{Q}$ is EXPTIME-complete, which is quite probable. Unfortunately, we were not able

to prove this conjecture and even show the decidability of ZIQ TBoxes. This is probably due to the fact that the decidability results for logics with inverses and counting are usually obtained via integer programming, which does not seem to be suitable for dealing with regular expressions.

The main idea of our reduction is to produce, from an input interpretation, an equivalent interpretation of high “girth”, so for the query it locally looks like a tree. For simplicity, the construction is presented in the ABox-free setting, but its slightly more complicated version works with ABoxes.

Theorem 3. *The finite conjunctive query entailment problem over \mathcal{L} TBoxes, where \mathcal{L} is any fragment ZIQ extending $ALCT_{Self}^{\cap}$, can be reduced in exponential time to its finite TBox satisfiability.*

The above theorem can be applied, *e.g.* to infer the decidability of finite CQ entailment for $ALCHIF$ extended with transitive closure of roles, see: [6].

For the future work we plan to take a closer look at the finite KBSAT for ZIQ and try to establish some results on finite entailment of non-local querying for the \mathcal{Z} family. A good idea to start with seems to be to consider transitive-closure enriched conjunctive queries, as done in [5].

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