Technische Universität Dresden

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# Formal Concept Analysis

Exercise Sheet 1, Winter Semester 2016/17

## 1 Set Theory

**Exercise 1** (a piece of recapitulation)

Given the following hints and the universe $M := \{1, 2, 3, 4, 5, 6, 7, 8\}$, compute the sets $A, B, C$:

$(a)$ $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

$(b)$ $B \cup C = \{1, 2, 4, 6, 8\}$

$(c)$ $A \cup C = \{1, 2, 3, 4, 5, 7, 8\}$

$(d)$ $A \cap B = \{2\}$

$(e)$ $B \cap C = \{2, 4, 8\}$

$(f)$ $A \cap C = \{2\}$

## 2 Logic

**Exercise 2** (repetition first-order logic)

Formalize the following statements for natural numbers $a, b, c$, using only multiplication ("$\cdot$"), equality ("$=$") and natural numbers ("0","1","2",...) besides the usual logical symbols ("$\neg$", "$\wedge$", "$\vee$", "$\rightarrow$", "$\leftrightarrow$", "$\forall$", "$\exists$", variables and parentheses):

$(i)$ $a$ divides $b$.

$(ii)$ $a$ is odd.

$(iii)$ $a$ is common divisor of $b$ and $c$

$(iv)$ $a$ is the gcd of $b$ and $c$.

$(v)$ $a$ is a square number.

$(vi)$ $a$ is a prime number.

## 3 Derivation Operators and Formal Concepts

**Exercise 3** (line diagram)

**a)** Recall: how is the derivation operator $(\cdot)'$ defined?

**b)** Let $\mathbb{K} = (G, M, I)$ be a formal context and let $A, B \subseteq G$. Prove the following statements:

1. $A \subseteq B$ implies $B' \subseteq A'$
2. $A \subseteq A''$
3. $A' = A'''$
4. For $C \in G$ and $D \in M$ holds: $(C, D)$ is a formal concept if and only if there is some $E \subseteq G$ such that $C = E''$ and $D = E'$.

# 4 Formal Concept Analysis

**Exercise 4** (Formal Context)

Regard the following formal context $\mathbb{K}$, given as a cross table:

| | needs water to live | lives in water | lives on land | needs chlorophyll to produce food | two seed leaves | one seed leaf | can move around | has limbs | suckles its offspring |
|---|---|---|---|---|---|---|---|---|---|
| Leech | x | x | | | | | x | | |
| Bream | x | x | | | | | x | x | |
| Frog | x | x | x | | | | x | x | x |
| Spike-Weed | x | x | | x | | x | | | |
| Reed | x | x | x | x | | x | | | |
| Bean | x | | x | x | x | | | | |
| Maize | x | | x | x | | x | | | |

**a)** Specify the following sets:

(*i*) $\{\text{Bean}\}'$

(*ii*) $\{\text{lives on land}\}'$

(*iii*) $\{\text{two seed leaves}\}''$

(*iv*) $\{\text{Frog, Maize}\}'$

(*v*) $\{\text{needs chlorophyll to produce food, can move around}\}'$

(*vi*) $\{\text{lives in water, lives on land}\}'$

(*vii*) $\{\text{needs chlorophyll to produce food, can move around}\}''$

**b)** Extend $\mathbb{K}$ with both an object and an attribute.