TECHNISCHE UNIVERSITÄT DRESDEN

Artificial Intelligence, Computational Logic

DECOMPOSING ABSTRACT DIALECTICAL FRAMEWORKS

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DRESDEN concept Exzellenz aus Wissenschaft und Kultur

Motivation

- Computational complexity of semantics for ADFs is in general higher than for AFs [Strass and Wallner, 2014].
- Algorithms based on SCC-recursive schema for AF semantics show significant performance gain [Cerutti et.al. KR 2014].
- We propose a similar approach based on a recursive decomposition along SCCs.
- Allows to define *cf2* and *stage2* semantics for ADFs.

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- Computational complexity of semantics for ADFs is in general higher than for AFs [Strass and Wallner, 2014].
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- We propose a similar approach based on a recursive decomposition along SCCs.
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Main Difference to AFs

1. Acceptance conditions of statements in sub-frameworks may still depend on statements not contained in sub-framework.
2. Elimination of redundancies from links and acceptance formulas.
3. Propagation of truth values to subsequent SCCs.

Agenda

ADFs - The Formal Framework

- Like AFs, use graph to describe dependencies among nodes.
- Unlike AFs, allow individual acceptance condition for each node.
- Assigns $\mathbf{t}$(rue) or $\mathbf{f}$(alse) depending on status of parents.

Definition

An abstract dialectical framework (ADF) is a tuple $D = (S, L, C)$ where

- S is a set of statements (positions, nodes),
- $L \subseteq S \times S$ is a set of links,
- $C = \{C_s\}_{s \in S}$ is a set of total functions $C_s : 2^{par(s)} \to \{\mathbf{t}, \mathbf{f}\}$, one for each statement s. C_s is called acceptance condition of s.

Semantics

Definition

Let φ be a propositional formula over vocabulary S and for an $M \subseteq S$ let $v : M \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$ be a three-valued interpretation.
The partial valuation of φ by v is $\varphi^v = \varphi[p/\mathbf{t} : v(p) = \mathbf{t}][p/\mathbf{f} : v(p) = \mathbf{f}]$.

Definition

Let $D = (S, L, C)$ be an ADF. A three-valued interpretation v is

- conflict-free iff for all $s \in S$ we have:
  - $v(s) = \mathbf{t}$ implies that φ_s^v is satisfiable,
  - $v(s) = \mathbf{f}$ implies that φ_s^v is unsatisfiable;
- naive iff it is $\leq_i$-maximal with respect to being conflict-free;

Where $\leq_i$ is a partial order over the truth values (resp. interpretations), i.e. $\mathbf{u} <_i \mathbf{t}$ and $\mathbf{u} <_i \mathbf{f}$.

Semantics ctd.

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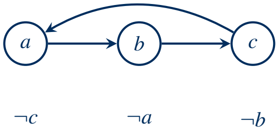
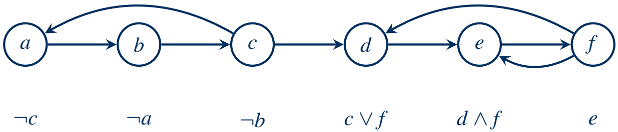
Example



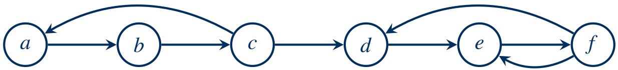
$v = \{a \mapsto \mathbf{f}, b \mapsto \mathbf{u}, c \mapsto \mathbf{t}\}$ is conflict-free, as $\varphi_a^v = \neg\mathbf{t}$ is unsatisfiable and $\varphi_c^v = \neg b$ is satisfiable.

Sub-Frameworks and Redundancies



- independent set $ind_D(\emptyset) = \{a, b, c\} = M_0$
- independent modulo M_0: $ind_D(M_0) = \{a, b, c, d, e, f\} = S$
- M independent set: sub-framework $D|_M = (M, L \cap (M \times M), \{\varphi_s\}_{s \in M})$

Sub-Frameworks and Redundancies



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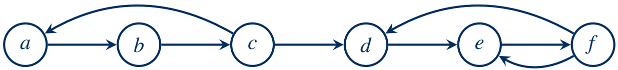
- Redundancies can change dependencies between statements.
- If (r, s) is redundant then r has no influence on the truth value of φ_s whatsoever.

Example

Consider $\varphi_s = a \vee (b \wedge c)$ and the interpretation $v = \{a \mapsto \mathbf{u}, b \mapsto \mathbf{f}, c \mapsto \mathbf{u}\}$.
$\varphi_s^v = a \vee (\mathbf{f} \wedge c)$

Sub-Frameworks and Redundancies



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$\varphi_s^v = a \vee (\mathbf{f} \wedge c) \equiv a$ c has no influence

Reduced ADF

Given an ADF $D = (S, L, C)$, an independent set $M \subseteq S$ and an interpretation $v : M \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$. The ADF D reduced with v on M is obtained by:

- adapt the acceptance condition of statement s to
  - $\mathbf{t}$ (resp. $\mathbf{f}$) if $v(s) = \mathbf{t}$ (resp. $v(s) = \mathbf{f}$)
  - $\neg s$ if $v(s) = \mathbf{u}$
  - partial valuation φ_s^v for remaining statements and if r is redundant in φ_s^v, replace r with $\mathbf{t}$
- remove redundant links
- add links $\{(s, s) \mid v(s) = \mathbf{u}\}$

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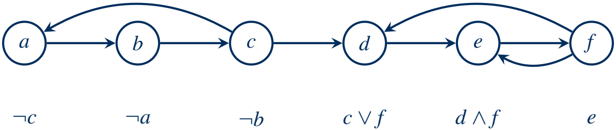
- remove redundant links
- add links $\{(s, s) \mid v(s) = \mathbf{u}\}$

Procedure

For a semantics σ and an ADF D, we obtain the σ_2 interpretations recursively by applying $\sigma_2(D) = \sigma_2(ind_D(\emptyset), D)$ by:

1. Start with all statements independent modulo $\emptyset$, i.e. $M_0 = ind_D(\emptyset)$
2. Compute all σ-interpretations of sub-framework $D|_{M_0}$
3. For each σ-interpretation w of $D|_{M_0}$ compute the reduced ADF
4. Call Step 1 with reduced ADF and $M_1 = ind_D(M_0)$.

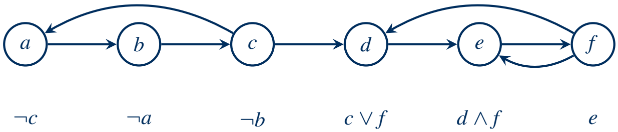
Example



$nai_2(D) = nai_2(ind_D(\emptyset), D)$

1. $ind_D(\emptyset) = \{a, b, c\} = M_0$

Example

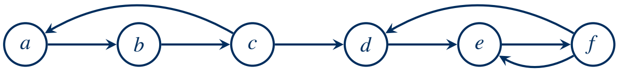


$nai_2(D) = nai_2(ind_D(\emptyset), D)$

1. $ind_D(\emptyset) = \{a, b, c\} = M_0$
2. Then we obtain $nai(D|_{M_0}) = \{v_0, v_1, v_2\}$:

$$
\begin{aligned}
v_0 &= \{a \mapsto \mathbf{u}, b \mapsto \mathbf{t}, c \mapsto \mathbf{f}\}, \\
v_1 &= \{a \mapsto \mathbf{f}, b \mapsto \mathbf{u}, c \mapsto \mathbf{t}\}, \\
v_2 &= \{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}, c \mapsto \mathbf{u}\}.
\end{aligned}
$$

Example



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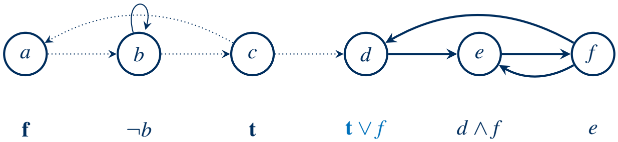

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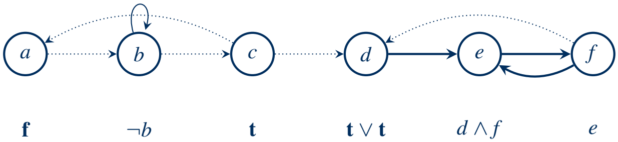
1. $ind_D(\emptyset) = \{a, b, c\} = M_0$
2. $v_1 = \{a \mapsto \mathbf{f}, b \mapsto \mathbf{u}, c \mapsto \mathbf{t}\}$
3. Reduced ADF with $\mathbf{t} \vee f \equiv \mathbf{t}$, thus link (f, d) is redundant

Example



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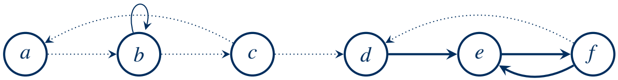
3. Reduced ADF

Example



$\mathbf{f}$ $\quad \neg b \quad$ $\mathbf{t}$ $\quad \mathbf{t} \vee \mathbf{t} \quad$ $d \wedge f$ $\quad e$

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1. $ind_D(\emptyset) = \{a, b, c\} = M_0$
2. $v_1 = \{a \mapsto \mathbf{f}, b \mapsto \mathbf{u}, c \mapsto \mathbf{t}\}$
3. Reduced ADF
4. Call Step 1 with reduced ADF D_1 and $M_1 = ind_{D_1}(M_0)$.

Example

$\mathbf{f}$ $\quad \neg b$ $\quad \mathbf{t}$ $\quad \mathbf{t} \vee \mathbf{t}$ $\quad d \wedge f$ $\quad e$

$nai_2(M_1, D_1)$

1. $M_1 = ind_{D_1}(M_0) = \{a, b, c, d\}$

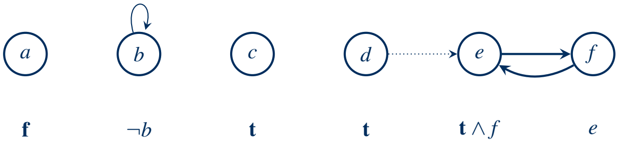
Example

$\mathbf{f}$ $\neg b$ $\mathbf{t}$ $\mathbf{t} \vee \mathbf{t}$ $d \wedge f$ e

$nai_2(M_1, D_1)$

1. $M_1 = ind_{D_1}(M_0) = \{a, b, c, d\}$
2. $nai(D|_{M_1}) = v_3 = v_1 \cup \{d \mapsto \mathbf{t}\}$

Example



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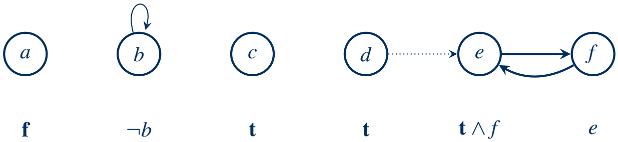

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2. $nai(D|_{M_1}) = v_3 = v_1 \cup \{d \mapsto \mathbf{t}\}$
3. Reduced ADF
4. Call Step 1 with reduced ADF D_2 and $M_2 = ind_{D_2}(M_1)$

Example

a	b	c	d	e	f
$\mathbf{f}$	$\neg b$	$\mathbf{t}$	$\mathbf{t}$	$\mathbf{t} \wedge f$	e

$nai_2(M_2, D_2)$

1. $M_2 = ind_{D_2}(M_1) = \{a, b, c, d, e, f\} = S$

Example

$\mathbf{f}$ $\quad \neg b$ $\quad \mathbf{t}$ $\quad \mathbf{t}$ $\quad \mathbf{t} \wedge f$ $\quad e$

$nai_2(M_2, D_2)$

1. $M_2 = ind_{D_2}(M_1) = \{a, b, c, d, e, f\} = S$
2. $nai(D_2) = \{v_4, v_6\}$:

$$v_4 = v_3 \cup \{e \mapsto \mathbf{t}, f \mapsto \mathbf{t}\}, \qquad v_6 = v_3 \cup \{e \mapsto \mathbf{f}, f \mapsto \mathbf{f}\}.$$

Main Theorem

Theorem

1. Let $\sigma \in \{\mathit{cfi}, \mathit{adm}, \mathit{pre}, \mathit{com}, \mathit{mod}\}$. Then $\sigma \leq \sigma_2$.
2. Let $\sigma \in \{\mathit{nai}, \mathit{stg}\}$. Then $\sigma \not\leq \sigma_2$.
3. Let $\sigma \in \{\mathit{cfi}, \mathit{nai}, \mathit{adm}, \mathit{pre}, \mathit{com}, \mathit{mod}\}$. Then $\sigma_2 \leq \sigma$.
4. Let $\sigma \in \{\mathit{stg}\}$. Then $\sigma_2 \not\leq \sigma$.

Conclusion

- We proposed a decomposition schema for semantics for ADFs.
- We introduced nai_2 and stg_2 for ADFs.
- Due to the relation of ADFs to logic programms we also get nai_2 and stg_2 semantics for LPs.
- Also in the paper: composing ADFs.

Future Work

- Analysis of the complexity of the approach.
- Implementation.
- Study splittings of ADFs and equivalences.

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