# Exercise Sheet 7: Query Optimisation and FO Query Expressivity
David Carral, Markus Krötzsch
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**Exercise 7.1.** Apply the conjunctive query minimisation algorithm to find a core of the following CQs:

1. $\exists x, y, z.\ R(x, y) \wedge R(x, z)$

2. $\exists x, y, z.\ R(x, y) \wedge R(x, z) \wedge R(y, z)$

3. $\exists x, y, z.\ R(x, y) \wedge R(x, z) \wedge R(y, z) \wedge R(x, x)$

4. $\exists v, w.\ S(x, a, y) \wedge S(x, v, y) \wedge S(x, w, y) \wedge S(x, x, x)$

**Exercise 7.2.** Consider a fixed set of relation names (each with a given arity). Show that there is a Boolean CQ $Q_{\min}$ without constant symbols that is most specific in the following sense:

For every BCQ $Q$ that does not use constants, we find that $Q_{\min} \sqsubseteq Q$.

Is there also a most general BCQ $Q_{\max}$ that contains all BCQs without constant names?

What is the answer to these questions if the considered BCQs may use constant names? What if we consider FO queries instead?

**Exercise 7.3.** Explain why the CQ minimisation algorithm is correct:

1. Why is the result guaranteed to be a minimal CQ?

2. Why is the result guaranteed to be unique up to bijective renaming of variables?

**Exercise 7.4.** For the following pairs of structures, find the maximal $r$ such that $\mathcal{I} \sim_r \mathcal{J}$:

(a)

(b)

(c)

(d)
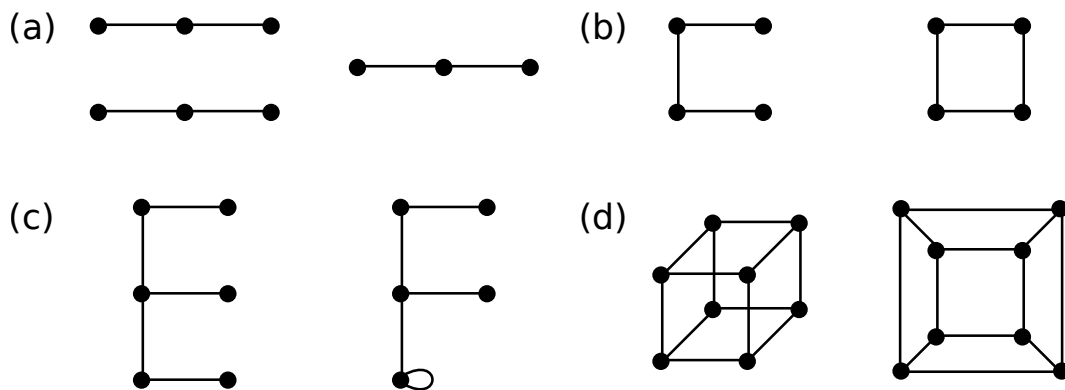
**Exercise 7.5.** A *linear order* is a relational structure with one binary relational symbol $\leq$ that is interpreted as a reflexive, asymmetric, transitive and total relation over the domain. Up to renaming of domain elements there is exactly one linear order for every finite domain, which can be depicted as a chain of elements. We denote the linear order of size $n$ by $\mathcal{L}_n$. For example:

$$\mathcal{L}_6 : 1 \leq 2 \leq 3 \leq 4 \leq 5 \leq 6 \qquad \text{and} \qquad \mathcal{L}_7 : 1 \leq 2 \leq 3 \leq 4 \leq 5 \leq 6 \leq 7$$

1. For which $r$ are $\mathcal{L}_6 \sim_r \mathcal{L}_7$?

2. More generally, for which $r$ are $\mathcal{L}_n \sim_r \mathcal{L}_{n+1}$?  $(*)$