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Complexity Theory

Exercise 8: Alternation

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Exercise 8.1. Describe a polynomial-time ATM solving **EXACT INDEPENDENT SET**:

Input: Given a graph G and some number k.
Question: Does there exists a maximal independent set in G of size exactly k?

Find a level of the polynomial hierarchy where this problem is contained in.

Exercise 8.2. Consider the Japanese game *go-moku* that is played by two players X and O on a 19x19 board. Players alternately place markers on the board, and the first one to have five of its markers in a row, column, or diagonal wins.

Consider the generalised version of go-moku on an $n \times n$ board. Say that a *position* of go-moku is a placement of markers on such a board as it could occur during the game. Define

$$\textbf{GM} = \{\langle B \rangle \mid B \text{ is a position of go-moku where X has a winning strategy}\}.$$

Describe a polynomial-time ATM solving **GM** and informally argue why this problem is not in any level of the polynomial hierarchy.

Exercise 8.3. Show that if $\mathsf{P} = \mathsf{NP}$, then $\mathsf{P} = \mathsf{PH}$.

Exercise 8.4. Show $\mathsf{NP}^{\mathsf{SAT}} \subseteq \Sigma_2\mathsf{P}$.

Exercise 8.5. Show the following result: *If there is any k such that $\Sigma_k^{\mathrm{P}} = \Sigma_{k+1}^{\mathrm{P}}$ then $\Sigma_j^{\mathrm{P}} = \Pi_j^{\mathrm{P}} = \Sigma_k^{\mathrm{P}}$ for all $j > k$, and therefore $\mathrm{PH} = \Sigma_k^{\mathrm{P}}$.*

Exercise 8.6. Show that $\mathrm{PH} \subseteq \mathrm{PSPACE}$.