Technische Universität Dresden

Prof. Dr. Sebastian Rudolph

Formal Concept Analysis Exercise Sheet 2, Winter Semester 2014/15

## 1 Lattice Theory

Exercise 1 (line diagram)

- a) Define: What is a lattice?
- b) Find a preferably small lattice and draw its line diagram.
- c) Which of the following line diagrams is not a lattice? Why?



Exercise 2 (complete lattice)

- a) Define: What is a complete lattice?
- b) Can you find a *complete* lattice among the lattices of Exercise 1c?
- c) Let  $P := (M, \leq)$  be an ordered set such that for every subset X of M the infimum  $\bigwedge X$  exists. Show that P is a complete lattice.

## Exercise 3

Prove the following theorem:

Let  $(L, \leq)$  be a lattice with supremum and infimum defined as usual. For any elements  $x, y, z \in L$  holds:

(i)  $x \wedge y = y \wedge x$ (ii)  $x \vee y = y \vee x$ (iii)  $x \wedge (y \wedge z) = (x \wedge y) \wedge z$ (iv)  $x \vee (y \vee z) = (x \vee y) \vee z$ (v)  $x \wedge (x \vee y) = x$ (vi)  $x \vee (x \wedge y) = x$ (vii)  $x \wedge x = x$ (viii)  $x \vee x = x$ 

**Exercise 4** (the first formal concepts)

Compute all formal concepts of the formal context shown in Table 1.

Tabelle 1: Grobian Gans: *Die Ducks. Psychogramm einer Sippe.* Rowohlt, Reinbek bei Hamburg 1972, ISBN 3-499-11481-X

|                 | generation |        |         | sex  |        | financial status |          |          |
|-----------------|------------|--------|---------|------|--------|------------------|----------|----------|
|                 | older      | middle | younger | male | female | rich             | carefree | indebted |
| Tick            |            |        | ×       | ×    |        |                  | ×        |          |
| Trick           |            |        | ×       | ×    |        |                  | ×        |          |
| Track           |            |        | ×       | ×    |        |                  | ×        |          |
| Donald          |            | ×      |         | ×    |        |                  |          | ×        |
| Daisy           |            | ×      |         |      | ×      |                  | ×        |          |
| Gustav          |            | ×      |         | ×    |        |                  | ×        |          |
| Dagobert        | ×          |        |         | ×    |        | ×                |          |          |
| Annette         | ×          |        |         |      | ×      |                  | ×        |          |
| Primus v. Quack | ×          |        |         | ×    |        |                  | ×        |          |