Exercise 7: Query Optimisation and First-Order Query Expressivity

Database Theory

2020-05-25

Maximilian Marx, David Carral

Exercise 1

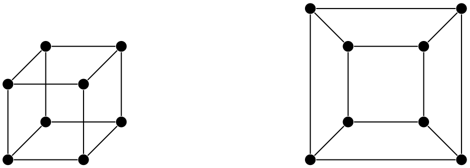
Exercise. For the following pairs of structures, find the maximal r such that $\mathcal{I} \sim_r \mathcal{J}$:

(i)

(ii)

(iii)

(iv)



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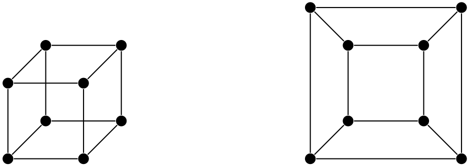

Solution.

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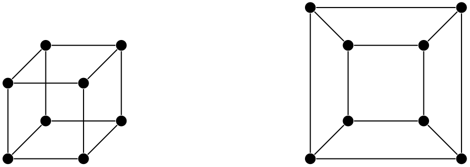

Solution.

(i) $r \leq 1$,

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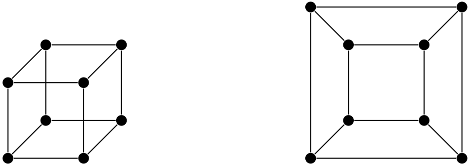

Solution.

(i) $r \leq 1$,

(ii) $r \leq 2$,

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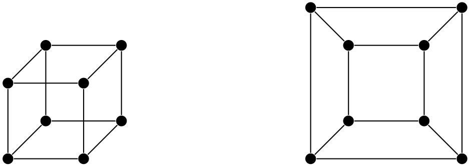

Solution.

(i) $r \leq 1$,

(ii) $r \leq 2$,

(iii) $r = 0$, and

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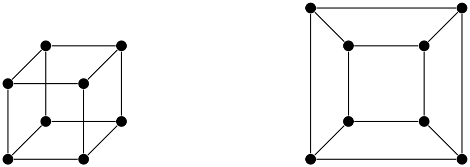

Solution.

(i) $r \leq 1$,

(ii) $r \leq 2$,

(iii) $r = 0$, and

(iv) $r \geq 0$.

Exercise 2

Exercise. A *linear order* is a relational structure with one binary relational symbol $\leq$ that is interpreted as a reflexive, asymmetric, transitive and total relation over the domain. Up to renaming of domain elements there is exactly one linear order for every finite domain, which can be depicted as a chain of elements. We denote the linear order of size n by $\mathcal{L}_n$. For example:

$$\mathcal{L}_6 : 1 \leq 2 \leq 3 \leq 4 \leq 5 \leq 6 \qquad \text{and} \qquad \mathcal{L}_7 : 1 \leq 2 \leq 3 \leq 4 \leq 5 \leq 6 \leq 7$$

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Theorem (11.10; Lecture 11, Slide 24)

The following are equivalent:

- $\mathcal{L}_m \sim_r \mathcal{L}_n$, *and*
- *either (1)* $m = n$, *or (2)* $m \geq 2^r - 1$ *and* $n \geq 2^r - 1$.

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Solution.

1. $r \leq 2$.
2. $n \geq 2^r - 1 \implies r \leq \lfloor \log_2(n+1) \rfloor$.