TU Dresden, Fakultät Informatik
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Complexity Theory

**Exercise 10: Randomised Computation**

17th January 2024

**Exercise 10.1.** Show that **MajSat** is in $\mathrm{PP}$.

$$\textbf{MajSat} = \{\varphi \mid \varphi \text{ is some propositional logic formula that is satisfied by more than half of its assignments}\}$$

**Exercise 10.2.** Show $\mathrm{BPP} = \mathrm{coBPP}$.

$*$ **Exercise 10.3.** Show $\mathrm{BPP}^{\mathrm{BPP}} = \mathrm{BPP}$.

**Exercise 10.4.** Find the error in the following proof that shows $\mathrm{PP} = \mathrm{BPP}$: *Let $\boldsymbol{L} \in \mathrm{PP}$. Then there exists a poly-time bounded PTM accepting $\boldsymbol{L}$ with error probability smaller than $\frac{1}{2}$. Using error amplification, we can make this error arbitrarily small, and in particular smaller than $\frac{1}{3}$. Hence, $\boldsymbol{L} \in \mathrm{BPP}$.*

**Exercise 10.5.** Let $\mathcal{M}$ be a polynomial-time probabilistic Turing machine. We say that $\mathcal{M}$ has *error probability smaller than* $\frac{1}{3}$ if and only if

$$Pr[\mathcal{M} \text{ accepts } w] < \frac{1}{3} \quad \text{or} \quad Pr[\mathcal{M} \text{ accepts } w] \geq \frac{2}{3}$$

for all inputs $w$. Show that deciding whether a polynomial-time probabilistic TM has error probability smaller than $\frac{1}{3}$ is undecidable.