Exercise 7.1. Show that the word problem of deterministic finite automata
\[ A_{DFA} = \{ \langle A, w \rangle \mid A \text{ a DFA accepting } w \} \]
can be decided in logarithmic space.

Exercise 7.2. Show that the composition of logspace reductions again yields a logspace reduction.

Exercise 7.3. Show that the word problem \( A_{NFA} \) of non-deterministic finite automata is NL-complete.

Exercise 7.4. Show that \( \text{BIPARTITE} = \{ \langle G \rangle \mid G \text{ a finite bipartite graph} \} \) is in NL. For this show that \( \text{BIPARTITE} \in \text{NL} \) and use \( \text{NL} = \text{coNL} \).

Hint: Show that a graph \( G \) is bipartite if and only if it does not contain a cycle of odd length.

Exercise 7.5. Say that a directed graph \( G \) is strongly connected if every two vertices in \( G \) are connected by a directed path. Show that \( \text{SC} = \{ \langle G \rangle \mid G \text{ a strongly connected graph} \} \) is NL-complete.

Hint: Provide a logspace reduction from reachability in directed graphs. You may want to add some edges to a graph \( G \) to make a path from \( s \) to \( t \) into a path from every vertex to every other vertex.

* Exercise 7.6. Show that checking whether an undirected graphs has a simple cycle can be done in logarithmic space.

Hint: If we perform a depth-first search on a vertex contained in a simple path, what will happen? Argue that depth-first search can be performed in logarithmic space.