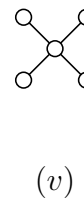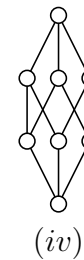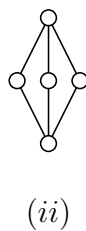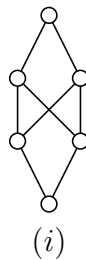# Formal Concept Analysis
Exercise Sheet 2, Winter Semester 2016/17

## 1 Lattice Theory

**Exercise 1** (line diagram)

  **a)** Define: What is a lattice?

  **b)** Find a preferably small lattice and draw its line diagram.

  **c)** Which of the following line diagrams does not represent a lattice? Why?



$(i)$        $(ii)$        $(iii)$        $(iv)$        $(v)$

**Exercise 2** (complete lattice)

  **a)** Define: What is a complete lattice?

  **b)** Can you find a *complete* lattice among the lattices of Exercise 1c?

  **c)** Let $P := (M, \leq)$ be an ordered set such that for every subset $X$ of $M$ the infimum $\bigwedge X$ exists. Show that $P$ is a complete lattice.

**Exercise 3**

Prove the following theorem:

Let $(L, \leqslant)$ be a lattice with supremum and infimum defined as usual. For any elements $x, y, z \in L$ holds:

  (i)  $x \wedge y = y \wedge x$            (ii)  $x \vee y = y \vee x$

  (iii)  $x \wedge (y \wedge z) = (x \wedge y) \wedge z$            (iv)  $x \vee (y \vee z) = (x \vee y) \vee z$

  (v)  $x \wedge (x \vee y) = x$            (vi)  $x \vee (x \wedge y) = x$

  (vii)  $x \wedge x = x$            (viii)  $x \vee x = x$