TECHNISCHE UNIVERSITÄT DRESDEN

Artificial Intelligence, Computational Logic

PROBLEM SOLVING AND SEARCH IN ARTIFICIAL INTELLIGENCE

Lecture 4 Tabu Search

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Dresden

DRESDEN concept
Exzellenz aus Wissenschaft und Kultur

Agenda

Tabu Search

Main Idea

- A memory forces the search to explore new areas of the search space
- Memorize solutions that have been examined recently. They become tabu points in next steps
- Tabu search is deterministic

Tabu Search and SAT

- SAT problem with $n = 8$ variables
- Initial (random) assignment $\mathbf{x} = (0, 1, 1, 1, 0, 0, 0, 1)$
- Evaluation function: weighted sum of number of satisfied clauses. Weights depend on the number of variables in the clause
- Maximize evaluation function (i.e. we're trying to satisfy all clauses)
- Random assignment provides $eval(\mathbf{x}) = 27$
- Neighborhood of $\mathbf{x}$ consists of 8 solutions. Evaluate them and select best
- **At this stage, it is the same as hill-climbing**
- Suppose flipping 3rd variable generates best evaluation ($eval(\mathbf{x}') = 31$)
- Memory keeps track of actions

Recency-based Memory

- Index of flipped variable + time when it was flipped
- **Differentiate between older and more recent flips**
- SAT: time stamp for each position of solution vector M (initialized to 0)
- Value of time stamp provides information on recency of flip at position

Memory Vector

$M(i) = j$ (when $j \neq 0$)
j is most recent iteration when i-th bit was flipped

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Assume information is stored for at most 5 iterations.

Alternative Interpretation

$M(i) = j$ (when $j \neq 0$)
i-th bit was flipped $5 - j$ iterations ago

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Example

0	0	5	0	0	0	0	0

Memory after one iteration. 3rd bit is **tabu** for next 5 iterations.

Different Interpretations

1st Variant

- Stores iteration number of most recent flip
- Requires a current iteration counter t which is compared with memory values
- If $t - M(i) > 5$ forget
- Only requires updating a single entry, and increase the counter
- **Used in most implementations**

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2nd Variant

- Values are interpreted as number of iterations for which a position is not available
- **All** nonzero entries are decreased by one at every iteration

Example ctd.

- Initial assignment $\mathbf{x} = (0, 1, 1, 1, 0, 0, 0, 1)$
- After 4 additional iterations M :

3	0	1	5	0	4	2	0

- Most recent flip $M(4) = 5$
- Current solution: $\mathbf{x} = (1, 1, 0, 0, 0, 1, 1, 1)$ with $eval(\mathbf{x}) = 33$

Example ctd.

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Neighborhood of $\mathbf{x}$

$$\mathbf{x}_1 = (0, 1, 0, 0, 0, 1, 1, 1) \qquad \mathbf{x}_5 = (1, 1, 0, 0, 1, 1, 1, 1)$$
$$\mathbf{x}_2 = (1, 0, 0, 0, 0, 1, 1, 1) \qquad \mathbf{x}_6 = (1, 1, 0, 0, 0, 0, 1, 1)$$
$$\mathbf{x}_3 = (1, 1, 1, 0, 0, 1, 1, 1) \qquad \mathbf{x}_7 = (1, 1, 0, 0, 0, 1, 0, 1)$$
$$\mathbf{x}_4 = (1, 1, 0, 1, 0, 1, 1, 1) \qquad \mathbf{x}_8 = (1, 1, 0, 0, 0, 1, 1, 0)$$

Example ctd.

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$$\mathbf{x}_3 = (1, 1, 1, 0, 0, 1, 1, 1) \qquad \mathbf{x}_7 = (1, 1, 0, 0, 0, 1, 0, 1)$$
$$\mathbf{x}_4 = (1, 1, 0, 1, 0, 1, 1, 1) \qquad \mathbf{x}_8 = (1, 1, 0, 0, 0, 1, 1, 0)$$

TABU, best evaluation $eval(\mathbf{x}_5) = 32$, **decrease!**

Example ctd.

- Current solution: $\mathbf{x} = (1, 1, 0, 0, 0, 1, 1, 1)$ with $eval(\mathbf{x}) = 33$
- New solution: $\mathbf{x}_5 = (1, 1, 0, 0, 1, 1, 1, 1)$ with $eval(\mathbf{x}_5) = 32$

3	0	1	5	0	4	2	0

changes to:

2	0	0	4	5	3	1	0

Example ctd.

- Current solution: $\mathbf{x} = (1, 1, 0, 0, 0, 1, 1, 1)$ with $eval(\mathbf{x}) = 33$
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changes to:

2	0	0	4	5	3	1	0

Policy might be too restrictive

- What if tabu neighbor $\mathbf{x}_6$ provides excellent evaluation score?
- Make search more flexible: override tabu classification if solution is outstanding

$\Longrightarrow$ **aspiration criterion**

Aspiration Criteria

Aspiration criteria are used to locally override tabu restrictions Some aspiration rules that we might add are:

- Aspiration by default: If all available moves are classified as tabu, and are not made admissible by some other AC, then the oldest move is selected.
- Aspiration by global form: Applies if the solution improves over the best so far solution
- Aspiration by influence: measures the degree of change of the new solution
  a) in terms of the distance between old and new solution
  b) change in solution's feasibility, if we deal with a constraint problem
  - Intuition: particular move has a larger influence if a larger step was made from old to new solution

Long-term Memory

Question

1. What is stored in long-term memory (think of SAT as an example)?
2. How can we escape local optima with help of a long-term memory?

Frequency-based Memory

- Operates over a longer horizon
- SAT: vector H serves as long-term memory.
  - Initialized to 0, at any stage of the search

$$H(i) = j$$

    interpreted as: during last h (horizon) iterations, the i-th bit was flipped j times
  - Usually horizon is large
  - After 100 iterations with $h = 50$, long-term memory H might have the following values

5	7	11	3	9	8	1	6

  - Shows distribution of moves throughout the last 50 iterations

Diversity of Search

Frequency-based memory provides information about which flips have been under-represented or not represented.
$\Longrightarrow$ we can diversify the search by exploring these possibilities

Use of Long-term Memory

Special Circumstances

- Situations where all non-tabu moves lead to worse solution
- To make a meaningful decision about which direction to explore next
- Typically: most frequent moves are less attractive
- Value of evaluation score is decreased by some penalty measure that depends on frequency, final score implies the winner

Example SAT

- Assume value of current solution is $eval(\mathbf{x}) = 35$
- Non-tabu flips $2, 3$ and 7 have values $30, 33, 31$
- None of tabu moves provides value greater than 37 (highest value so far)
  $\implies$ we can't apply aspiration criterion

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- None of tabu moves provides value greater than 37 (highest value so far) $\Longrightarrow$ we can't apply aspiration criterion
- Frequency based-memory and evaluation function for new solution $\mathbf{x}'$ is

$$eval(\mathbf{x}') - penalty(\mathbf{x}')$$

- $penalty(\mathbf{x}') = 0.7 \times H(i)$, where 0.7 coefficient, $H(i)$ value from long-term memory H :

7	for solution created by flipping 2nd bit
11	for solution created by flipping 3nd bit
1	for solution created by flipping 7nd bit

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- New scores are:

$30 - 0.7 \times 7 = 25.1$	2nd bit
$33 - 0.7 \times 11 = 25.3$	3nd bit
$31 - 0.7 \times 1 = 30.3$	7th bit

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Diversify Search

Including frequency values in a penalty measure for evaluating solutions.

Tabu Search and the TSP

- Move: swap two cities in a particular solution
- Current solution: $(2, 4, 7, 5, 1, 8, 3, 6)$
- 28 neighbors $\binom{8}{2} = \frac{7 \cdot 8}{2} = 28$
- Recency-based memory: swap of cities i and j in i-th row and j-th column (for $i < j$)
- Maintain number of remaining iterations for which swap stays on tabu list
- Frequency-based memory: same structure; indicate totals of all swaps within horizon $h = 50$

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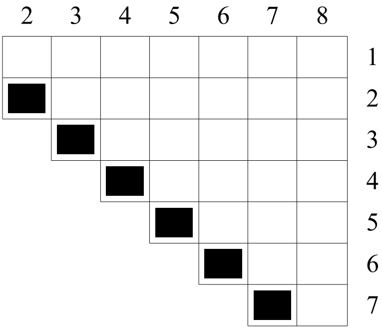

Tabu Search and the TSP ctd.

- Assume both memories initialized to zero and 500 iterations have been completed
- Current solution: $(7, 3, 5, 6, 1, 2, 4, 8)$ with length: 173, best solution so far 171

2	3	4	5	6	7	8	
0	0	1	0	0	0	0	1
■	0	0	0	5	0	0	2
	■	0	0	0	4	0	3
		■	3	0	0	0	4
			■	0	0	2	5
				■	0	0	6
					■	0	7

2	3	4	5	6	7	8	
0	2	3	3	0	1	1	1
■	2	1	3	1	1	0	2
	■	2	3	3	4	0	3
		■	1	1	2	1	4
			■	4	2	1	5
				■	3	1	6
					■	6	7

left: recency-based memory; right: frequency-based memory

Summary

- Simulated annealing and tabu search are both design to escape local optima
- Tabu search makes uphill moves only when it is stuck in local optima
- Simulated annealing can make uphill moves at any time
- Simulated annealing is stochastic, tabu search is deterministic
- Compared to classic algorithms, both work on complete solutions. One can halt them at any iteration and obtain a possible solution
- Both have many parameters to worry about

References

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